The Divider Set: A New Concept in Morphology

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Abstract: - A new concept in the area of morphology is introduced. It is a generalization of the thinning and skeletonization concepts. It can be used in most applications where skeletons or Voronoi sets are utilized, having specific advantages over the traditional results due to its rigorous mathematical definition. A set of rules for the construction of the innovative form of skeleton are presented, based on a mathematical description of the construction process. Certain examples are given, with an eye to specific applications, such as robotic navigation or OCR, among others.

Keywords: - Divider, skeleton, lattice, morphology, OCR, navigation of robots.

1 Introduction

The concepts of thinning, skeletons and the Voronoi sets are useful in many fields of morphology, such as image processing, OCR, or navigation of robotic devices. Their application is limited only by the difficulties and ambiguities in their construction methods [1-5, 7-10, 13].

The concept of the Divider of a set A, symbolized by Div(A), does not have most of the limitations and drawbacks of the above state-of-the-art concepts. It is defined by a set of equations and inequalities, solvable through computer calculations. As a result, its construction is free of most difficulties existing in the “burned grass” or “maximal disks” algorithms. In this preliminary work, a simplified form of the definition and construction process of the Divider concept is presented by the authors. Further results, concerning precise mathematical definitions, descriptions of the algorithms for the construction of the Divider both by equation solving and by discrete lattice process, described in principle here, development and improvement of the utilized software and possible applications, will be presented in next papers.

Preliminary results show promise in two areas of interest to the authors. One is Optical Character Recognition [1-6, 10-12], where the advantages of the Divider over previous forms of character skeletons are expected to provide unambiguous features of classification and recognition, along with important points of graph ical information. The other is navigation of autonomous robotic agents in a highly complex, enclosed environment, with the help of a topological map similar to those in use in bus and metro lines [13].

2 Preliminary concepts, fundamental definitions.

Definition 1.1. Let there be a finite lattice of square cells, where each cell has eight neighbors (Fig. 2d). Each pair of successive neighbors defines a straight line on the lattice. Straight lines may be horizontal (Fig. 1a), vertical (Fig. 1b), left diagonal (Fig. 1c) and right diagonal (Fig 1d).

Definition 1.2. The eight straight lines defined by a cell and each of its eight neighbors are called the directions starting from the specific cell. The directions are numbered with the numbers 0 to 7, starting from the upper left direction and proceeding clockwise (Fig. 2).

The directions numbered 1, 3, 5, 7 are called main directions. The directions numbered 0, 2, 4, 6 are called secondary directions.

Postulate 1.1 Any arithmetic operations executed with the numbers of the 8 neighbor directions will follow the rules of arithmetic mod(8), unless distinctly otherwise specified.

Definition 1.3. An angle is made of two straight lines passing through a cell. The cell is then called the apex of the angle (Fig. 3). The angle of Fig. 3b is also called a two dimensional formation of three cells.

Definition 1.4. A set of cells is called one dimensional at a cell if there are at most two straight lines passing through the specific cell of the set and if the specific cell is not a part of a two dimensional subset of three cells (Fig. 3b). A set of cells is called two dimensional in a cell if there are more than two straight lines through the specific cell or if the specific cell is a part of a two dimensional subset of three cells (Fig. 3b). A set of cells is called one-dimensional iff it is one-dimensional at all its cells (Fig. 3a).

Lemma 1.2. A two dimensional set of cells contains three or more cells.

Lemma 1.3. An angle containing a two dimensional formation of three cells plus a fourth cell, may be transformed into a one-dimensional angle by removing the apex cell (Fig 3b and 3c).

Lemma 1.4. A formation of three consecutive cells is one dimensional if only the middle one is in contact with the two others and if the numbers of the directions of the neighbors differ by 2 or more.

Proof. If the numbers differ by 1, the neighbors are adjacent and there is a two dimensional formation of three cells.

Definition 1.5. A cell belonging to a set of cells is called an interior cell iff the neighbors 1, 3, 5, 7, in the main directions, belong to the set. A cell of the set having at least one of its main neighbors belonging to the complement of the set is called a boundary cell of the set.

Lemma 1.5 A set must contain at least 5 cells so that one of them may be an interior cell.

Proof. An interior cell must have at least 4 neighbors attached to its main directions, belonging to the same set.

3 The definition of the Contact Square and The Divider.

The best way to introduce a metric to a finite sized two-dimensional lattice is to use the square metric. It is defined for every pair of cells C_1, C_2, having coordinates (x_1, y_1) and (x_2, y_2) by the formula:
$D(C_1, C_2) = \max\{| x_1 - x_2|, | y_1 - y_2|\}$. (1).

All sets of cells obviously have boundary cells. A set of cells also having interior cells has a boundary surrounding the interior cells in such a way so that there is no path leading from an interior cell to a cell of the complement of the set without passing through the boundary. This, intuitively obvious but actually very difficult to prove result, is known to topologists as a very important theorem due to Jordan.

The concepts of the interior of a set of cells, its boundary and the existence of loops and cavities are very important in morphology applications. In Optical Character recognition (OCR), the existence, number, shape and relative position of loops plays a crucial role. If a set of cells does not contain a loop enclosing a cavity, traditional methods of thinning work only in the sense of creating a simplified drawing of a shape.

In contrast, in the definition of the Divider set, the existence of loops, interior points or cavities is irrelevant.

Definition 2.1 Let a set of cells A be considered. Let its boundary be defined. At each boundary cell, there are neighbors belonging to the complement of the set. The directions from each boundary cell towards these neighbors are called free directions. Furthermore, if a direction is free and if the adjacent directions have numbers differing from the initial direction by 1, then the initial direction is called a contact direction of the specific boundary cell.

As an example, if direction 0 is free in a boundary cell and directions 1 and 7 are also free, direction 0 is a contact direction.

Lemma 2.1 If a square disk has as its center a neighbor of a boundary cell and radius 1, the intersection of the disk with the set A will contain only boundary cells, both of the disk and of the initial set.

Proof. The center of the disk is the only interior cell of the disk. It is not included in the intersection with A, since it belongs to the complement. The radius of the disk is 1, while the distance from any interior cell will be more than 1, since a path from the center to an interior cell will have a boundary cell to pass. Therefore, the intersection contains only boundary cells of A and the disk.

If, instead of the neighbor of the boundary set, the next cell of the complement and a disk of radius 2 having it as center are considered, the intersection of the disk with the set may or may not contain interior cells. The larger disk will naturally contain the former one. This process may be repeated until one of two things happens: Either the limits of the available lattice are reached or the intersection of the disk with A will contain interior cells of A.

Definition 2.2 The largest disk, which has its center along the contact direction of a boundary cell of A and its intersection with A includes only boundary cells both of the disk and A, is called a maximal disk (Fig. 4).

Definition 2.3 A maximal disk is called a contact disk if at least one of the following happens (Fig. 4).

1. The intersection of A with the disk contains boundary cells of A forming an angle.

2.The intersection of A with the maximal disk is disconnected.

3. None of the above happens, but the next largest disk in the same contact direction fulfills at least one of the conditions 1 and 2. Then, the larger disk not being a maximal disk, the intersection of A with it will contain interior cells of both A and the disk.

If one of the above happens, the maximal disk is called a contact disk and its center is called a contact center.

Corollary 2.1 Conditions 1 and 2 of Definition 2.3 may be true together. If condition 3 is true, then both 1 and 2 are not valid.

Definition 2.4 The set of all contact centers in relation to a set A is called the Divider of A and is symbolized as Div (A).

4 The process of construction of the Divider of a set of cells.

The construction of the Divider of any set A of cells may start at any boundary cell belonging to A. The steps to be followed are described below:

Step 1: All boundary cells of A are considered as a separate cell, symbolized as Bound(A).

Step 2: The neighbor directions are examined and the contact directions are marked.

Step 3: The successive disks with centers along each contact direction and radii increasing by one at each step are considered. If one of conditions 1, 2 or 3 are satisfied or if the limit of the lattice is reached, the process stops.

Step 4:. If a contact center is defined by step 3, it is marked as a cell belonging to Div(A). Then the process goes on to step 5. If, on the other hand, the process is finished by reaching the limit of the lattice, it continues with step 5.

Step 5: The next cell of Bound(A) is considered and steps 1 to five are again taken, until all cells of Bound(A) are processed. The resulting set Div A) is a mathematical generalization of both the skeleton of a set with interior cells and the Voronoi set of the set. Its utilization in all kinds of applications of skeleton like figures and relative methods are obvious.

5 Discussion.

The results of the previous process on certain rather simplified sets are partly presented in Figs 6 and 7. The difficulties anticipated concerned certain fundamental properties desired in any skeleton construction. The resulting graphs should have simply connected branches. In other words they should not contain loops or parallel attached lines and they should not contain interior points in the sense described above. The parts of the Divider contained within closed loops of the initial set should not have more than one simply connected component. First examples presented here have been processed by a simple thinning and unifying algorithm, which took care of any disconnections or multiple connections. The only cases of two dimensional formations with interior points, or double consecutive lines going on to the end of the lattice, appeared in exterior branches of the Divider, proceeding to the limit of the lattice. These examples, mostly not shown here, due to lack of space, occur in a few and easily described formations in initial sets and are characterized by very distinct symmetry features.. Therefore, the rules for correcting such defects in the Divider are very straightforward and easy to apply. A simple example is given in Fig 6 (unprocessed Divider) and Fig 7 (processed Divider).

In navigation applications [13], where the initial sets are expected to be enclosed and the Divider branches going on to the end of the lattice should not be included in a navigation map, such features are of no importance. In OCR [6], due to the characteristics of the Hellenic and Latin characters, the connected components of the Divider of a character outside any loops included in its figure are of the greater importance. In that case, the authors' research results so far indicate that simple optimizing algorithms, based on combinatorics (also see [1], p. 69), will yield the solution to any minor problems.

6 Conclusions.

The paper presents the concept of Divider of a set, with the associated definitions and lemmas. The concepts have been implemented through appropriate algorithms, which have been tested extensively through a wide set of examples. Further work on the construction of the Divider both by equation solving and by discrete lattice process, will be presented in the next set of papers.

References:

[1] Tsang Ing Jyh Pattern Recognition, Neighborhood Codes, and Lattice Animals (2000) Ph.D Thesis,Univ. of Antwerp, Holland. http://citeseer.ist.psu.edu/jyh00pattern.html

[2] I. J. Tsang, I. R. Tsang, B. De Boeck and D. Van Dyck, "Scaling and criticalprobability for cluster size and lattice animals diversity on randomly occupied square lattices", Journal of Physics A: Mathematical and General, Vol. 33, Nr. 14, p. 2739-2754, (2000).

[3] Maher Ahmed Rabab Ward, "A Rotation Invariant Rule-Based Thinning Algorithm for Character Recognition", IEEE Transactions on Pattern Analysis and Machine Intelligence December 2002 (Vol. 24, No. 12) pp. 1672-1678.

[4] George Nagy, Thomas A. Nartker, Stephen V. Rice, "Optical Character Recognition: An illustrated guide to the frontier" Procs. Document Recognition and Retrieval VII, SPIE Vol. 3967, 58-69 (invited)

[5] Ning Li, "An implementation of ocr system based on skeleton matching" Technical Report 3-93*, University of Kent, Computing Laboratory, University of Kent, Canterbury, UK, March 1993.

[6] S.J. Perantonis, B. Gatos, K. Ntzios, I. Pratikakis, I. Vrettaros, A.S. Drigas, C. Emmanouilidis, A. Kesidis, D. Kalomirakis, "A System for Processing and Recognition of Old Greek Manuscripts (The D-SCRIBE Project)", *WSEAS Transactions on Computers*, Issue 6, Volume 3, pp. 2049-2057, 2004.

[7] D.Kalles & D.T.Morris, "A Novel Fast and Reliable Thinning Algorithm" Image and Vision Computing, Vol. 11, 1993, pp. 588-603.

[8]L.Hayat, A.Naqvu & M.B.Sandler, "Comparative evaluation of Fast Thinning Algorithms on a Multiprocessor Architecture". Image and Vision Computing, Vol.10, 1992, pp. 210-218.

[9] C.Neusius & J.Olszewski, "A Noniterative Thinning Algorithm" ACM Transactions on Mathematical Software, Vol.20, 1994, pp. 5-20.

[10] S.Impedovo, L. Ottaviano & S.Occhiegro, "Optical Character Recognition – A Survey",International Journal of Pattern Recognition and Artificial Intelligence, Vol. 5, 1991, pp. 1-24.

[11] J.J.Hull, "A Database for Handwritten Text Recognition Research", IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 16, 1994, pp. 550-554.

[12] G.Kim & V.Gorivindaraju, "A Lexicon Driven Approach to Handwritten Word Recognition for Real-Time Applications", IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 19, 1997, pp. 366-379.

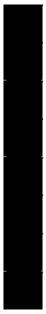
[13] G. Sakellariou, M. Shanahan and B. Kuipers.Skeletonization as mobile robot navigation. Proc. Towards Autonomic Robotic Systems (TAROS-04,) 2004, pages 149–155.

Fig. 1

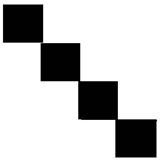
a to d: straight lines

b

a



c



d

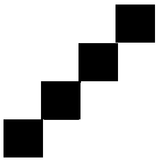
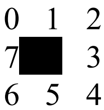
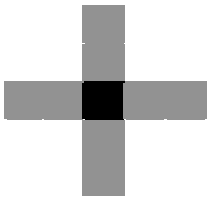


Fig.
2

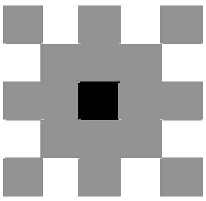
d Neighbors of a cell



b: Main directions



a: All directions



c: Secondary directions

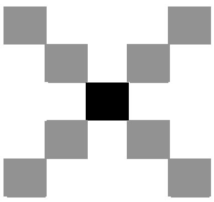
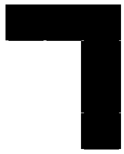


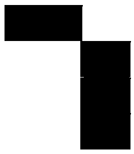
Fig. 3

a:

b:



c;



d

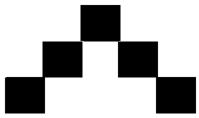


Fig 4

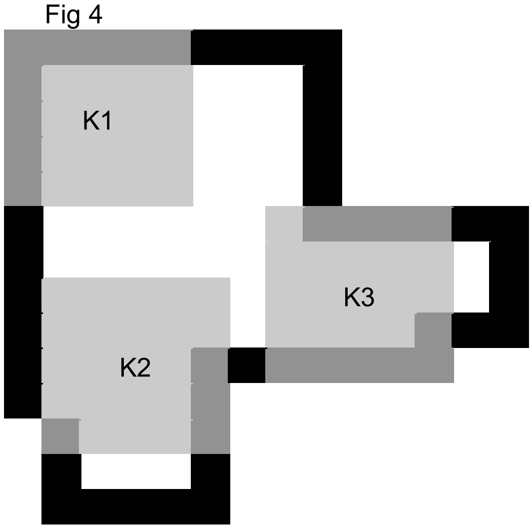


Fig 5

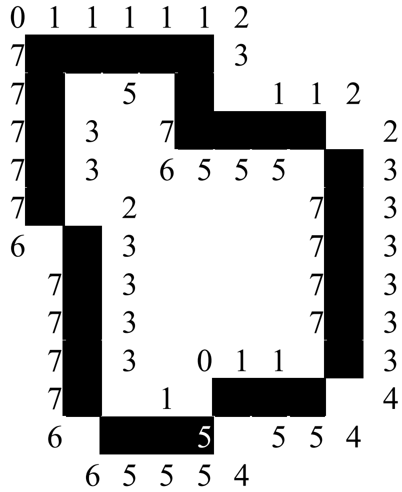


Fig 6

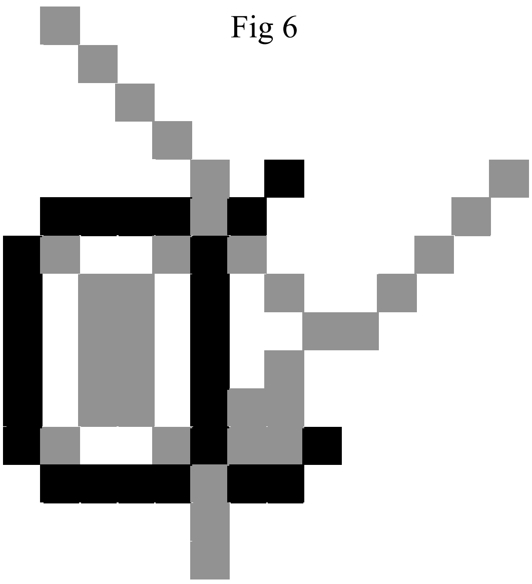


Fig 7