Indications of Chaos in Walgraef-Aifantis Model

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Abstract

In this work we explore Walgraef – Aifantis model for dislocation diffusion simulation, using a technique we describe in the text. Our approach produces significant results and indications of chaotic behaviour near the area of $\alpha = 0.8$, as explained in our model.

Introduction

The Walgraef – Aifantis model is a gradient dependent model for studying the dislocation dynamics of cyclic deformation.
In its simplest form [WA85], [ZA99], the model is described by the equations:

$$\frac{\partial \rho_m}{\partial t} = D_m \frac{\partial^2 \rho_m}{\partial x^2} + \beta\,\rho_i - \gamma \rho_m \rho_i^2 \tag{1}$$

$$\frac{\partial \rho_i}{\partial t} = D_i \frac{\partial^2 \rho_i}{\partial x^2} + (g-\beta)\,\rho_i - \gamma \rho_m \rho_i^2 \tag{2}$$

In the simplest one dimensional form of the model, the mobile dislocations density ρ_m and the immobile dislocations density ρ_i are functions of a space parameter x and a time parameter t.
The parameters β and γ are phenomenological coefficients [WA85] and D_m and D_i, $D_i \ll D_m$ [ZA99], are diffusion coefficients.
From considerable experimental and theoretical evidence, indications of fractal and possibly chaotic behavior have been found.

Form of the Solutions

In this presentation, a specific form of solutions is studied and indications of chaos are demonstrated. The behavior of the system of solutions depends strongly on the values of the system parameters. The solutions of Equations 1 and 2 are supposed to be expandable in series with respect to the time parameter t:

$$\rho_m = \sum_{j=0}^{N} y_j\, t_j \tag{3}$$

$$\rho_i = \sum_{k=0}^{N} z_k\, t_k \tag{4}$$

Where the y_j and the z_k are functions of x. By taking derivatives and equating the coefficients of equal powers of t in 1 and 1, the following expressions are obtained for the y_j and the z_k:

$$y_N + 1 = D_m \frac{,d^2 y_N}{dx^2} + \beta\, z_N - \gamma\, A_N \tag{5}$$

$$z_N + 1 = D_m \frac{d^2 z_N}{d_x^2} + (g-\beta) z_N + \gamma\, A_N \tag{6}$$

A_N is an auxiliary function defined as follows:

$$A_N = \sum_{i=0}^{N} \left(y_i \sum_{j=0}^{N-i} z_j\, z_N - i - j \right) \tag{7}$$

The first few A_N are calculated:

$$\begin{aligned} A_0 &= y_0\, z_0^2 \\ A_1 &= 2\, y_0\, z_0\, z_1 + y_1\, z_0^2 \\ A_2 &= y_0 (2\, z_0\, z_2 + z_1^2) + 2\, y_1\, z_0\, z_1 + y_2\, z_0^2 \\ A_3 &= 2\, y_0\, (z_0\, z_3 + z_1\, z_2) + y_1\, (2\, z_0\, z_2 + z_1^2) + 2\, y_2\, z_0\, z_1 + y_3\, z_0^2 \end{aligned}$$

By choosing appropriate boundary conditions y_0 and z_0, calculating their second derivatives with respect to x and inserting them into the first set equations, the iterative calculation of the series defining the mobile dislocations density ρ_m and the immobile dislocations density ρ_i is initiated.

Initial Conditions

We explored several functions as initial configuration of the distribution of immobile dislocations. Our initial approach was a Trigonometrical function $\cos(x)$.

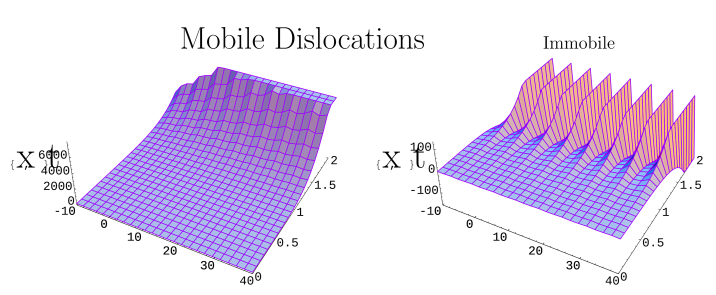

FIGURE 1: We can clearly see the distribution of both mobile and immobile dislocations. Is clearly indicated that the model does exhibits a sudden deviation approximately 4 seconds after tension is applied.

On the Edge of Chaos

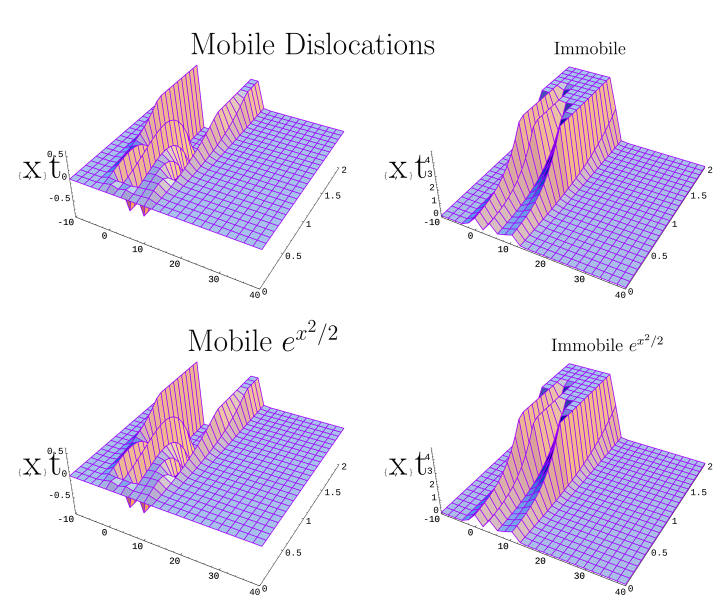

FIGURE 2: Moreover we used a the initial configuration a more sophisticated distribution of Gaussian peaks of the form $\rho_i^0 = \sum_{i=0}^{N} e^{-\lambda\left(\frac{(x-i\eta)^2}{2}\right)^{\alpha}}$, were in this particular model we used $\lambda = 1$, and $\eta = 5$ for the correlation distance between two peaks.

No Correlation Distribution

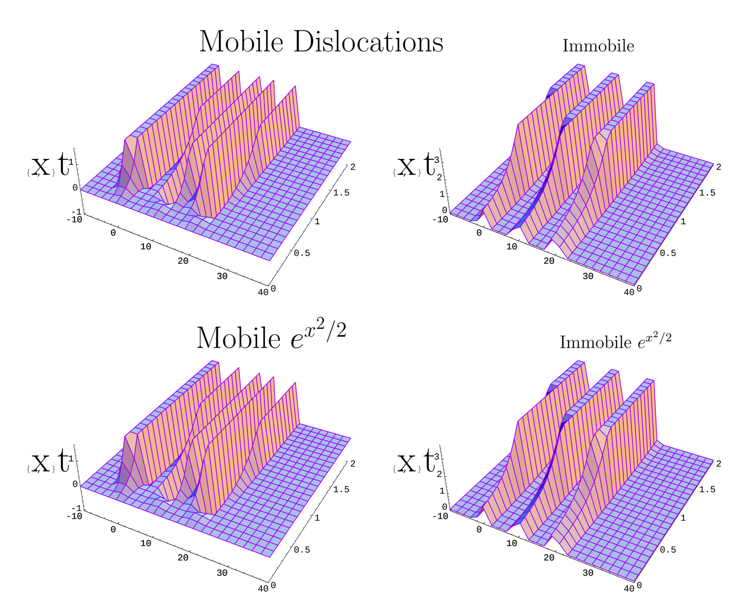

FIGURE 3: We Plot exactly the same configuration only this time for η >correlation length. The model has been tested for its convergence, which is obtained after 6 iterations (Quadratic Convergence. We also note that best behaviour was observed near $\alpha \simeq 0.8$

Conclusions

We explored the system 1 2 of partial differential equations, with an alternative method, and obtained triggering results concerning the dynamics governing the dislocation diffusion system.
In future work we intend to explore more analytically the chaotic behaviour of the model using sophisticated tools as Bifurcation theory, mathematical treatment of chaotic maps and analytically Transfer Operators.

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