Indications of Chaos in Walgraef-Aifantis Model

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Abstract

In this work we explore Walgraef-Aifantis model for dislocation diffusion simulation, using a technique we describe in the text. Our approach produces significant results and indications of chaotic behaviour near the area of $\alpha = 0.8$, as explained in our model.

Introduction

The Walgraef-Aifantis model is a gradient dependent model for studying the dislocation dynamics of cyclic deformation. In its simplest form, the model is described by the equations:

$$\frac{\partial \rho}{\partial t} = D_\alpha \frac{\partial^2 \rho}{\partial x^2} + (y - \beta)\rho - \gamma \rho^2$$

$$\frac{\partial \rho}{\partial t} = D_\beta \frac{\partial^2 \rho}{\partial y^2} + (x - \gamma)\rho - \gamma \rho^2$$

In the simplest one-dimensional form of the model, the mobile dislocations density $\rho_m$ and the immobile dislocations density $\rho_i$ are functions of a space parameter $x$ and a time parameter $t$. The parameters $\beta$ and $\gamma$ are phenomenological coefficients, and $D_\alpha$ and $D_\beta$ are diffusion coefficients. From considerable experimental and theoretical evidence, indications of fractal and possibly chaotic behavior have been found.

Form of the Solutions

In this presentation, a specific form of solutions is studied and indications of chaos are demonstrated. The behavior of the system of solutions depends strongly on the values of the parameters. The solutions of Equations 1 and 2 are supposed to be expandable in series with respect to the time parameter $t$.

$$\rho_{\alpha} = \rho_0 e^{-\beta t}$$

$$\rho_{\beta} = \rho_0 e^{-\gamma t}$$

Where $\rho_0$ and $\rho_0$ are functions of $x$. By taking derivatives and equating the coefficients of equal powers of $t$ in 1 and 2 the following expressions are obtained for the $\rho_i$.

$$\rho_{\alpha} + 1 = D_\alpha \frac{\partial^2 \rho_{\alpha}}{\partial x^2} + \beta x \rho_{\alpha} - \gamma \rho_{\alpha}^2$$

$$\rho_{\beta} + 1 = D_\beta \frac{\partial^2 \rho_{\beta}}{\partial y^2} + (y - \beta) \rho_{\beta} + \gamma \rho_{\beta}^2$$

$A_{\alpha}$ is an auxiliary function defined as follows:

$$A_{\alpha} = \sum_{n=0}^{N} \left( \frac{\partial}{\partial \alpha} \sum_{j=0}^{n} \left| j \rho_{\alpha} - i - j \right| \right)$$

The first few $A_{\alpha}$ are calculated:

$A_0 = \rho_0 e^{-\beta t}$

$A_1 = \rho_0 (\rho_0 e^{-\beta t} + 2 \rho_0 e^{-\beta t} \frac{\partial \rho_0}{\partial t} + \beta \rho_0 - \gamma \rho_0^2)$

$A_2 = \rho_0 \left( 2 \rho_0 e^{-\beta t} + \beta \rho_0 - \gamma \rho_0^2 \right) + 2 \rho_0 \frac{\partial \rho_0}{\partial t} (\rho_0 e^{-\beta t} + \beta \rho_0 - \gamma \rho_0^2) + \gamma \rho_0^2 e^{-\beta t}$

By choosing appropriate boundary conditions $\rho_0$ and $\rho_0$ calculating their second derivatives with respect to $x$ and inserting them into the left set equations, the iterative calculation of the series defining the mobile dislocations density $\rho_m$ and the immobile dislocations density $\rho_i$ is initialized.

Conclusion

We explored the system of partial differential equations, with an alternative method, and obtained triggering results concerning the dynamics governing the dislocation diffusion system. In future work we intend to explore more analytically the chaotic behavior of the model using sophisticated tools as bifurcation theory, mathematical treatment of chaotic maps and analytically Transfer Operators.

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References
