An experimental study of helicon resonance in metals

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An experimental study of helicon resonance in metals

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Abstract

This review is devoted to helicons—electromagnetic waves propagating in the electron–hole plasma in metals at low temperatures. The review is divided into two parts. In the first part we present the theory of, and the experimental data on, helicons in the absence of effects caused by Landau quantisation of conduction electron energy in a strong magnetic field. We emphasise the peculiar features of helicon propagation which are due to the anisotropy of the electron spectrum of metals, effects not observable in a gaseous plasma. An analysis of data on the anisotropy of the collision damping, the Landau damping and on dopplerons—a new branch of electromagnetic excitations in the vicinity of the Doppler-shifted cyclotron resonance—is given. We also discuss the results of studies of the acoustic satellites of helicon resonance and the peculiarities of helicon propagation in the intermediate state of type-I superconductors. In the second part we present the results of observations of new effects caused by Landau quantisation, namely oscillations of the phase velocity of helicons, non-linear helicon resonance and the anomalous attenuation of helicons in metals with diamagnetic domains. Examples are discussed in which the helicon resonances are employed to investigate transport relaxation in high magnetic fields, the production of defects by plastic deformation, open orbits and the absolute amplitude of the de Haas–van Alphen effect.

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1. Introduction

The 1960s gave rise to a new, fruitful and exciting field in solid-state physics—the electrodynamics of weakly damped low-frequency electromagnetic waves in metals. Its origin goes back to the theoretical paper by Konstantinov and Perel' (1960), in which it was predicted that circularly polarised electromagnetic waves can propagate in a metal in a strong magnetic field. In view of their polarisation such waves were christened 'helicons' (Aigrain 1960). A year later audio-frequency helicons were observed in pure sodium samples cooled to liquid helium temperatures and placed in a uniform constant magnetic field of the order of 10 kG (Bowers et al. 1961).

The discovery of helicons attracted considerable attention to the subject as the possibility of weakly damped electromagnetic waves in metals was quite unexpected. Previously it had been assumed that electromagnetic waves with frequencies below the plasma frequency, typically $10^{15}$ s$^{-1}$ for conventional metals, would be attenuated in the thin surface layer and the classical skin effect was a mental block to searches for the low-frequency electromagnetic wave phenomena in metals. The discovery of helicons was followed by discoveries of many other kinds of electromagnetic waves in metals: the Alfvén and fast magnetosonic waves (Kirsch 1963, Khaikin et al. 1963), cyclotron waves (Walsh and Platzman 1965), spin waves in non-ferromagnetic metals (Schultz and Dunifer 1967), dopplerons (Fisher et al. 1971) and thermomagnetic and galvanomagnetic waves (Kopylov 1978, 1979).

A second source of interest in helicons arose from the striking features of the helicon wave itself. Helicons propagate very slowly; their phase velocity being smaller by a factor of $10^{5}$–$10^{10}$ than the velocity of light in vacuum. From the point of view of electrodynamics, the metals in which helicons can propagate are reminiscent of transparent dielectrics with a refractive index $N = 10^{5}$–$10^{9}$. A plane-parallel metallic plate may act like, for example, a Fabry–Perot resonator. To excite the resonances of helicon waves in the submillimetre range it is sufficient to employ an alternating electromagnetic field of frequency ~100 Hz in a magnetic field of 1 kG (see figure 8).

Since the phase velocity of helicons is so small, they may resonantly interact with waves of other kinds and/or quasi-particles, such as conduction electrons (Taylor et al. 1963a, b), acoustic waves (Grimes and Buchsbaum 1964), magnons (Grimes 1966) and surface-type acoustic waves (Petrashov 1979). Furthermore, strong non-linear electromagnetic effects were discovered in the propagation of helicons by Bozhko and Volskii (1977a, b).

A further stimulus for the study of helicons has been their use in practical investigations of the properties of metals. Of many applications of helicon resonance we cite the electrodeless Hall effect measurements (Chambers and Jones 1962), measurements of the Fermi surface parameters (e.g. Hui 1969, Giovanelli and Merrill 1970), investigations of magnetic breakdown (Delaney 1974), measurements of the absolute amplitude of the de Haas–van Alphen effect (Volskii and Petrashov 1970) and investigations of conduction electron scattering (e.g. Volskii and Petrashov 1975).

The experimental results of the first decade have been summarised in the excellent reviews by Morgan (1967), Kaner and Skobov (1968) and Maxfield (1969). The
purpose of the present review is to take account of the very considerable activity during the intervening years.

During the last decade the development in the subject has indeed been very rapid. It is characterised by the following.

(i) Much more detailed data on the propagation of helicons in metals with anisotropic Fermi surfaces have become available. This has led to the discovery of qualitatively new phenomena and mechanisms which have no analogue in a gaseous plasma. In the early studies it was quite common to stress the analogy between helicons in metals and the so-called 'atmospheric whistlers', the circularly polarised waves in a gaseous plasma. Nowadays such an analogy is quite unhelpful, for even in such a 'simple' metal as potassium, in which the helicon propagation laws are most similar to those for a gaseous plasma, a number of 'anomalies' are observed.

(ii) The understanding of the influence of quantum effects on helicon propagation has almost completely changed (§§ 4 and 5).

(iii) The successful production of ultra-high-purity metals has led to advances in the studies of metals.

(iv) Exact solutions of the electromagnetic boundary value problem for the propagation of helicons in finite-size samples have been obtained (§ 2.3).

(v) The understanding of the macroscopic acousto-helicon phenomena, which strongly affect the results of measurements based on the helicon resonance, has substantially been improved (§ 3.2).

2. Theory of helicons in the absence of quantum effects

The propagation of helicons, which are circularly polarised electromagnetic waves, in the high-conductivity medium of a metal is possible because of the existence of the Hall effect. One can refer to helicons as the dynamic manifestation of the Hall effect.

Before deriving the proper wave equations and calculating the basic characteristics of helicons, a qualitative discussion of those phenomena which affect their propagation is in order. Consider an electromagnetic wave with electric field \( \mathbf{e} \) perpendicular to a

![Figure 1](image_url)

Figure 1. Typical trajectories of electrons in crossed electric and magnetic fields in metals with a closed Fermi surface in momentum space \((p\text{ space})\) (top) and real space \((r\text{ space})\) (bottom). (a) Spherical FS (alkali metals, free electron gas), (b) example of anisotropic FS (indium, aluminium, lead; second Brillouin zone), (c) model FS of a compensated metal with vanishing total Hall current.
uniform magnetic field. Provided that the mean free path of electrons is sufficiently large and the frequency of the wave is low enough, the electrons would drift in the direction normal to the plane formed by the uniform magnetic field and the electric field of the wave, see figure 1(a). This drift of electrons induces the Hall current. The important feature of the Hall current is that it is non-dissipative. It does not result in Joule losses because the current is perpendicular to the electric field:

\[ Q = j_H \cdot e = 0. \]  

(2.1)

Hence in the sequence of phenomena which lead to the wave process in a metal—induction of the electric current by virtue of the Ohm–Hall law, induction of the magnetic field by virtue of the Ampère law and induction of the electric field by virtue of the Faraday law, etc—there are no dissipative processes. Hence the electromagnetic energy is conserved and once created the wave does not attenuate.

The situation in metals with an anisotropic closed Fermi surface (FS)† (figure 1(b)) is qualitatively the same: the crossed electric and magnetic fields induce a non-dissipative Hall drift. Only the shape of the electron orbit changes.

For an open FS the situation changes qualitatively (figure 2(a)). Let the opening be directed in real space not perpendicularly to the electric field. The motion of electrons on open orbits results in a dissipative current and attenuation of the wave. For sufficiently large numbers of electrons in open orbits the propagation of helicons is impossible.

\[ F_{\text{2.}1} \]

\[ P_{\text{2.}1} \]

Figure 2. Illustration of the origin of the dissipative component of the drift of electrons along the electric field in crossed electric and magnetic fields. Top: p space; bottom: r space. (a) Open FS with opening in real space not perpendicular to the electric field, (b) collision-induced dissipative drift, (c) drift in a non-uniform electric field (schematic), (d) origin of open trajectories due to magnetic breakdown in aluminium. P is the magnetic breakdown probability.

One more special case is that of compensated metals, when the FS is comprised of electron and hole parts with an equal number of electrons and holes. In such a system the Hall current in a strong magnetic field is negligibly small, as both holes and electrons drift in the same direction and there is no charge transfer (figure 1(c)). Hence no propagation of helicons.

The propagation of helicons is strongly affected by the scattering of conduction electrons on lattice imperfections, impurity atoms, lattice vibrations, etc. Collisions

† For a discussion of the dynamics of electrons in metals with a complex FS see, for example, the book by Cracknell and Wong (1973).
result in abrupt variations of the phase of the circular motion of electrons in the magnetic field. Such discontinuities of the phase lead to a drift of the orbit centre along the electric field of the wave (figure 2(b)), i.e. to a dissipative current and the consequent attenuation of helicons. A low helicon attenuation therefore demands that the Hall current is much larger than the dissipative one.

The Hall current is a non-dissipative one because, while moving in the uniform electric field (figures 1(a) and (b)), the electron accelerates during the first half of the cyclotron period and decelerates during the second half, such that the mean electron-wave energy exchange vanishes. However, in the non-uniform field of the wave the acceleration and deceleration do not compensate each other exactly (figure 2(c)). This results in a dissipation of the energy of the wave, which can be expressed in terms of the dissipative current (figure 2(c)). This current is a function of the electric-field variation along the path of electrons between two successive collisions. The above discussion exemplifies the attenuation induced, not by collisions of electrons or open orbits, but rather by the spatial variation of the electric field of the wave.

We conclude that the propagation of helicons in metals strongly depends on the FS topology, in particular the presence of open orbits, the relative hole–electron concentration, the conduction electron collision frequency and the variation of the electric field of the wave along the path of electrons between successive collisions.

Whether helicons will propagate or not depends on the macroscopic Hall current, whatever its origin. For instance, in the type-II superconductors and in the intermediate state of the type-I superconductors the Hall effect is of much more complex origin than in normal metals. Nevertheless the helicon-like electromagnetic waves can propagate (Maxfield and Johnson 1965, Druyvesteyn et al 1966).

We now turn to a detailed quantitative description of the propagation of helicons in normal metals.

2.1. Dispersion relation

A quantitative description of electromagnetic waves in metals is obtained using Maxwell's equations together with the equation which relates the electric field of the wave e to the current density j:

\[ \text{curl } e = -\frac{1}{c} \frac{\partial b}{\partial t} \]  
(2.2)

\[ \text{curl } h = \frac{4\pi}{c} j \]  
(2.3)

\[ j = j(e). \]  
(2.4)

The displacement current \((4\pi)^{-1}(\partial e/\partial t)\) is neglected, as is common in the case of metals, being much smaller than the conduction current. Since this implies \(\text{div } j = 0\), the deviations from electroneutrality are neglected. We also put \(h = b\), supposing the metal to be non-magnetic with a susceptibility close to unity. In a sufficiently strong magnetic field Landau quantisation leads to a strongly oscillating dependence of \(h\) on \(b\), thus drastically changing the electrodynamic properties of the metal. The related phenomena will be discussed below in §§4 and 5. In the absence of quantum effects, all the properties of the electromagnetic wave depend on the detailed form of \(j\) in (2.4) as a function of \(e\).
In general, the current density $j(r, t)$ is a non-local function of the electric field with spatial and time coherence of the order of the electron mean free path and collision time, respectively (e.g. Kaner and Skobov 1968).

The simplest case is for fields which vary slowly over distances of the order of the mean free path and during the time between collisions. In this case the current $j$ is related to the field $e$ by the generalised Ohm’s law:

$$j(r, t) = \hat{\sigma} e(r, t).$$

(2.5)

Here $\hat{\sigma}$ is the static conductivity tensor, which depends on the properties of the metal, on the external magnetic field, the temperature and the concentration of impurities and imperfections, etc, but not on the frequency and wavelength of the wave. For obvious reasons the relationship (2.5) is called the local relationship. It holds provided that

$$\omega \tau \ll 1 \quad (2.6)$$

$$kl \ll 1 \quad (\lambda \gg l) \quad (2.7)$$

where $\lambda$ and $\omega$ are the wavelength and frequency of the wave, $k = 2\pi / \lambda$ is the wavenumber, $l$ is the mean free path and $\tau$ is the relaxation time of the electrons.

The first of the above inequalities, the condition for the wave being time-dispersion-free (no retardation effects), is practically always fulfilled for the frequencies of interest, $\omega < 10^8$ s\(^{-1}\), since even in metals of the highest purity currently available and at the lowest temperatures $\tau$ does not exceed $10^{-8}$ s.

It is much more difficult to fulfil the condition (2.7) for the spatial-dispersion-free wave. On the one hand, because the Fermi velocity of electrons is large, $v_F = 10^8$ cm s\(^{-1}\), the mean free path is fairly large: a relaxation time of $\tau = 10^{-9}$ s corresponds to $l = 1$ mm. On the other hand, the phase velocity of helicons is very small, $v_H = (10^{-5} - 10^{-9}) c$ (c being the velocity of light), so that the relatively short wavelengths can be excited even at very low frequencies.

If inequalities (2.6) and (2.7) are not fulfilled, the current is related to the electric field via the complex integral relationship (see, for example, Kaner and Skobov (1968)). Fortunately, in metals this relationship is practically always the linear one. Hence in terms of Fourier transforms one has the algebraic equation:

$$j_n(k, \omega) = \sigma_{nm}(k, \omega) e_m(k, \omega) \quad (n, m = x, y, z).$$

(2.8)

Sometimes it is more convenient to invert equation (2.8) and to employ the resistivity tensor $\rho$, which enables one to express the electric field $e$ in terms of the current:

$$e_i = \rho_{ik} j_k \quad (i, k = x, y, z).$$

(2.9)

These formulae can be greatly simplified if written down for the coordinate axes properly oriented relative to the wave propagation direction. However, in a specific problem, one has normally to use the components of the conductivity tensor for the given metal, since the current–field relationship is calculated either using the kinetic equation (see, for example, Kaner and Skobov (1968)) or some other approximation (e.g. Chambers 1980) and one calculates the response of the electron system to the given electric field and expresses it in terms of the conductivity tensor.

One of the purposes of this section is to relate parameters of the helicons to the microscopic parameters of the metal and to its $\mathbf{FS}$. Perhaps losing some simplicity, we shall write down all the formulae in terms of conductivity in the coordinate system.
Figure 3. Polarisation of helicons. Coordinate systems $OXYZ$ and $OX'Y'Z'$ are connected with the directions of the external magnetic field $B_0$ and the propagation direction of helicons $k$, respectively. The electric field of the wave $e$ is, in general, elliptically polarised in a plane $OXY$, whereas the magnetic field $b$ and current are polarised in a plane $OX'Y'$, the polarisation being almost circular.

with the $z$ axis directed along the constant magnetic field (figure 3(a)). For the plane wave with wavevector $k$:

$$ e, h \sim \exp [i(kr - \omega t)]. \quad (2.10) $$

Using Maxwell’s equations (2.2) and (2.3) to eliminate the magnetic field we obtain

$$ j = \frac{ic^2}{4\pi \omega} [k \times (k \times e)]. \quad (2.11) $$

Equation (2.11) implies that the current is always perpendicular to the wavevector $k$. Relating $j$ to the field $e$ via the tensor relationship (2.8), one gets a system of homogeneous linear equations:

$$ \hat{\sigma} e = \frac{ic^2}{4\pi \omega} [k \times (k \times e)]. \quad (2.12) $$

In the coordinate system chosen $k = k(0, \sin \theta, \cos \theta)$ (see figure 3(a)) and (2.12) can be written as

$$ [\sigma_{xx} + (ic^2 k^2/4\pi \omega)]e_x + \sigma_{yx}e_y + \sigma_{xz}e_z = 0 \quad (2.13(a)) $$

$$ \sigma_{yx}e_x + [\sigma_{yy} + (ic^2 k^2/4\pi \omega)]e_y + [\sigma_{yz} - (ic^2 k_z/4\pi \omega)]e_z = 0 \quad (2.13(b)) $$

$$ \sigma_{zx}e_x + [\sigma_{zy} - (ic^2 k_z/4\pi \omega)]e_y + [\sigma_{zz} + (ic^2 k^2/4\pi \omega)]e_z = 0. \quad (2.13(c)) $$

The relevant secular equation results in the dispersion equation, which can be written down in the following form (Halevi and Rabinovich 1970):

$$ c^2 k^2 / 2\pi \omega = (\beta / \alpha) \{ \pm [(4\alpha \gamma / \beta^2) - 1]^{1/2} + i \} \quad (2.14(a)) $$

where

$$ \alpha = \sigma_{zz} \cos^2 \theta + \sigma_{yy} \sin^2 \theta + (\sigma_{yz} + \sigma_{zy}) \sin \theta \cos \theta \quad (2.14(b)) $$

$$ \beta = \sigma_{xx} (\sigma_{xx} \cos^2 \theta + \sigma_{yy}) - (\sigma_{xy} \sin \theta + \sigma_{xz} \cos \theta) \times (\sigma_{yx} \sin \theta + \sigma_{zx} \cos \theta) \quad (2.14(c)) $$

$$ + \sigma_{xx}(\sigma_{yy} + \sigma_{xy}) \sin \theta \cos \theta \quad (2.14(c)) $$

$$ \gamma = -\sigma_{zz} \sigma_{yy} - \sigma_{yy} \sigma_{zz} - \sigma_{xy} \sigma_{yz} - \sigma_{yx} \sigma_{zy} \quad (2.14(d)) $$

These expressions are general and can be reduced to the simpler ones in specific cases.
2.1.1. Metals with spherical Fermi surfaces—the local limit. Inserting into the dispersion equation (2.14(a)) the zero-magnetic-field static conductivity yields a strongly damped electromagnetic wave. Indeed, in the absence of a magnetic field the conductivity is scalar, i.e. the conductivity tensor is the diagonal one with only dissipative components:

\[ \hat{\sigma} = \sigma_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

\[ \sigma_0 = n e^2 \tau / m \] (2.15)

where \( n \) is the charge carrier concentration, \( m \) is the mass of carriers and \( e \) is the modulus of the elementary charge. The parameters \( \alpha, \beta \) and \( \gamma \) in (2.14(b))–(2.14(d)) take the simpler form:

\[ \alpha = \sigma_0 \]
\[ \beta = 2\sigma_0^2 \]
\[ \gamma = \sigma_0^3. \] (2.16)

Using these, (2.14(a)) reduces to the well-known wavelength–frequency relationship for the normal skin effect:

\[ c^2 k^2 / 2\pi\omega = 2\sigma_0 i \]
\[ k = (2\pi\sigma_0 \omega / c^2)^{1/2}(1+i). \] (2.17)

The wave is essentially attenuated in the skin depth \( \delta \):

\[ \delta = \frac{1+i}{k} = c(2\pi\sigma_0 \omega)^{-1/2}. \] (2.18)

The magnetic field has a striking influence on the conductivity (see, for example, Kittel (1963)):

\[ \hat{\sigma} = \sigma_0 \begin{pmatrix} 1/[1+(\omega_c \tau)^2] & \mp \omega_c \tau/[1+(\omega_c \tau)^2] & 0 \\ \pm \omega_c \tau/[1+(\omega_c \tau)^2] & 1/[1+(\omega_c \tau)^2] & 0 \\ 0 & 0 & 1 \end{pmatrix}. \] (2.19)

Here \( \omega_c = eB_0 / mc \) is the cyclotron frequency, \( B_0 \) is the constant magnetic field and the signs ‘+’ and ‘−’ correspond to holes and electrons, respectively.

The parameter \( \omega_c \tau \) is a measure of the strength of the magnetic field. It corresponds roughly to the number of revolutions of an electron during the time interval between two successive collisions. In the weak magnetic field, \( \omega_c \tau \ll 1 \), the conductivity is dissipative and the skin effect persists. In the strong magnetic field, \( \omega_c \tau \gg 1 \), the situation is reversed. Non-dissipative components are much larger than the dissipative ones, \( \sigma_{xy}, \sigma_{yx} \gg \sigma_{xx}, \sigma_{yy} \), with the exception of \( \sigma_{zz} \), which retains its magnitude and is much larger than all the other components of \( \hat{\sigma} \).

Retaining in (2.14(b))–(2.14(d)) the leading terms in \( 1/\omega_c \tau \), we obtain

\[ \alpha = \sigma_0 \cos^2 \theta \]
\[ \beta = 2\sigma_0^2 (\omega_c \tau)^{-2} \] (2.20)
\[ \gamma = \sigma_0^3 (\omega_c \tau)^{-2}. \]

Further, if

\[ 4\alpha \gamma / \beta^2 = (\omega_c \tau)^2 \cos^2 \theta \gg 1 \] (2.21)
the dispersion equation (2.14(a)) has two solutions:

\[ c^2 k^2 / 2 \pi \omega = \pm 2(\gamma / \alpha)^{1/2} + i(\beta / \alpha). \] (2.22)

The first one, with ‘+’ in (2.22), corresponds to a weakly damped helicon wave, whereas the second one corresponds to strong attenuation.

The final form of the dispersion relation for helicons in metals with a spherical FS is then

\[ (\text{Re} \ k)^2 = \frac{4 \pi n e \omega}{c B_0 \cos \theta}. \] (2.23)

Note that helicons have a quadratic spectrum, \( \omega \sim k^2 \), which depends only on the charge carrier concentration, rather than the parameters of the electron energy spectrum, such as velocities and masses. Sometimes it is convenient to rewrite (2.23) in terms of the Hall constant \( R_H \):

\[ (\text{Re} \ k)^2 = \frac{4 \pi \omega}{c^2 R_H B_0 \cos \theta}. \] (2.24)

To find the polarisation of the wave one has to solve equations (2.13(a))–(2.13(c)) and compute \( e_z / e_x \) and \( e_y / e_x \). Taking into account (2.21) and keeping only the leading terms in \( 1 / \omega_c \tau \), one finds

\[ e_z / e_x = -c / (\omega_c \tau) \tan \theta \] (2.25)

\[ e_y / e_x = \pm i / \cos \theta. \] (2.26)

In strong fields \( e_z / e_x \ll 1 \), so that the helicon electric field lies in a plane almost normal to the magnetic field.

If \( \theta = 0 \), the components \( e_x \) and \( e_y \) have phases differing by \( \pi / 2 \), corresponding to circular polarisation. The relative sign in (2.26) depends on the charge of the carriers. The polarisation rotates in the same sense as the charge carriers along their orbits in the magnetic field. For \( \theta \neq 0 \) the polarisation is an elliptic one (figure 3(a)). In the attenuated wave, corresponding to the sign ‘−’ in (2.22), the electric field rotates in the reverse direction. Whatever the propagation direction, the projection of the helicon electric field onto a plane, normal to the propagation direction, has a circular polarisation. By virtue of Maxwell’s equations the magnetic field of the wave lies in the plane normal to the propagation direction and has a circular polarisation (figure 3(b)).

It is interesting to estimate the characteristic parameters of helicons in metals having a typical carrier concentration of \( n \approx 10^{22} \text{ cm}^{-3} \).

The phase velocity

\[ \nu_H = \frac{\omega}{\text{Re} \ k} = (c B_0 \omega \cos \theta / 4 \pi n e)^{1/2} \] (2.27)

changes in the range from 1 to \( 10^5 \text{ cm s}^{-1} \) for magnetic fields 1–100 kG and frequencies \( f = \omega / 2 \pi = 1\text{–}10^8 \text{ Hz} \). Such low velocities correspond to a refraction index of \( N = c / \nu_H = 10^2\text{–}10^9 \). A consequence of this is that, irrespective of the angle of incidence of the exciting electromagnetic wave, helicons propagate practically perpendicular to the surface of a metal. The corresponding wavelength varies from \( 10^3 \) to \( 10^{-3} \text{ mm} \).

Now we can estimate the impurity concentration at which helicons can propagate in fields of 1–100 kG.
Helicon resonance in metals

In terms of the on-impurity scattering cross section $\langle \sigma \rangle$ and the impurity concentration $N_{\text{imp}}$ the mean free path is

$$l = 1/N_{\text{imp}}(\sigma).$$

(2.28)

If the scattering on impurities is the dominant subprocess, the helicons can propagate, provided that

$$\omega_c \tau = \omega_c (v_F N_{\text{imp}}(\sigma))^{-1} \gg 1$$

(2.29)

where $v_F$ is the Fermi velocity of electrons. The resulting upper bound for the impurity concentration is

$$N_{\text{imp}} \ll eB_0 / mc v_F(\sigma).$$

(2.30)

For $m = m_0$ ($m_0$ is the free electron mass), $v_F = 10^8$ cm s$^{-1}$, $\langle \sigma \rangle = a^2$, where $a$ is the lattice parameter ($a = 4 \times 10^{-8}$ cm). Then (2.30) implies $N_{\text{imp}} \ll 10^{14} B_0$ (cm$^{-3}$).

Thus the notion of the ‘strong magnetic field’ is a relative one. In metals of the highest available purity the strong-field criterion, $\omega_c \tau \gg 1$, is well fulfilled already in magnetic fields of the order of 100 G (Krylov et al. 1974) and helicons can propagate, whereas in high-concentration alloys or pure metals at high temperatures even the strongest available magnetic fields may prove ‘weak’.

Consider now a more general case with non-local effects and an anisotropic FS. The problem the experimentalist encounters is that the locality condition depends on the mean free path, so that a priori it is unknown whether the experimental conditions correspond to the local situation or not. Rewriting (2.23) as

$$\omega < cB_0 / 4 \pi n e^2 |\cos \theta|$$

(2.31)

then if the mean free path $l = 1$ mm and $B_0 = 1$ kG, the onset of non-local effects corresponds to a frequency $f \approx 10$ kHz. The threshold frequency rises steeply as the mean free path decreases. These effects are conveniently discussed in terms of the dimensionless parameter $kR$, where $R = v_F / \omega_c$ is the Larmor radius. The influence of non-local effects on the spectrum and damping of helicons differs drastically in the long-wavelength, $kR \ll 1$, and short-wavelength, $kR \approx 1$ regimes. In the non-local regime the Doppler effect becomes important. Indeed, for an electron with average velocity $v_0$ the effective frequency of the electromagnetic field is

$$\omega_D = \omega + (kv_0).$$

(2.32)

If the frequency $\omega_D$ in (2.32) proves to be equal to the cyclotron frequency, resonance can take place. This phenomenon is referred to as Doppler-shifted cyclotron resonance (DSCR). Such a resonance interaction turns out to be so strong that the resulting absorption of the wave energy by electrons destroys the helicon. In the vicinity of the resonance the helicon spectrum is substantially modified (McGroddy et al. 1966, Overhauser and Rodriguez 1966).

The DSCR criterion is

$$\omega + (kv_0) = \omega_c.$$ 

(2.33)

In a sufficiently strong magnetic field one has $k_z v_{0,\text{max}} + \omega < \omega_c$, so that there is no resonance absorption of the wave. In view of $\omega < \omega_c$ the DSCR criterion can be rewritten as $k_z v_{0,\text{max}} / \omega_c = kR \approx 1$. Therefore the long-wavelength limit, $kR \gg 1$, corresponds to the experimental conditions far away from an absorption ‘edge’, when the influence of DSCR is negligible. It should be emphasised that the criterion for different regimes does not contain the electron mean free path.
2.1.2. Metals with arbitrary closed Fermi surfaces—long waves: $kR \ll 1$. In a strong magnetic field the conductivity tensor (2.8) can be written in the form (e.g. Kaner and Skobov 1968):

$$
\sigma_{ik}(k, \omega) = \begin{vmatrix}
    a_{xx}/B_0^2 & a_{xy}/B_0 & a_{xz}/B_0 \\
    a_{yx}/B_0 & a_{yy}/B_0^2 & a_{yz}/B_0 \\
    a_{zx}/B_0 & a_{zy}/B_0 & a_{zz}
\end{vmatrix}
$$

(2.34)

where the quantities $a_{ik}$ depend on the FS topology, the relaxation time of the conduction electrons and, if the locality condition does not hold, on the parameters of the wave.

It is noteworthy that in the long-wavelength limit the Hall components of (2.34) do not depend on the parameters of the wave and are functions only of the difference of the electron and hole concentrations $n_e$ and $n_h$:

$$
\sigma_{xy} = -\sigma_{yx} = -\frac{(n_e - n_h)ec}{B_0}.
$$

(2.35)

The dispersion relation for helicons in a strong magnetic field can be derived in a similar manner to before using formulae (2.14). Retaining the leading terms in $1/B_0$, we obtain

$$
\alpha = \sigma_{zz} \cos^2 \theta
$$

$$
\beta = \sigma_{zz}(\sigma_{xx} \cos^2 \theta + \sigma_{yy}) + (\sigma_{yx} \sin \theta + \sigma_{zx} \cos \theta)^2 + \sigma_{xy}^2
$$

$$
\gamma = \sigma_{zz} \sigma_{yy}.
$$

(2.36)

If in (2.35) $n_e \neq n_h$ (uncompensated metal), then in a sufficiently strong magnetic field,

$$
4\alpha \gamma / \beta^2 \gg 1
$$

(2.37)

and one can use (2.22). The resulting dispersion relation takes the form

$$
(\Re k)^2 = \frac{4 \pi \omega}{c^2 |\cos \theta| |\sigma_{yx}|} = \frac{4 \pi |n_e - n_h| e\omega}{cB_0 |\cos \theta|}.
$$

(2.38)

Hence in metals with a closed FS of arbitrary shape the spectrum of helicons does not depend on the FS topology; rather, it is a function of the electron and hole concentrations.

The formulae (2.25) and (2.26) for the wave polarisation are replaced by

$$
e_z/e_x = -(\sigma_{yx} \sin \theta + \sigma_{zx} \cos \theta \pm i\sigma_{zy}) / \sigma_{zz} \cos \theta
$$

(2.39)

$$
e_y/e_x = \pm i / \cos \theta.
$$

(2.40)

In view of (2.34) one readily finds that $e_z/e_x \sim \text{constant}/B_0 \ll 1$. Hence, in strong magnetic fields, irrespective of the shape of the closed FS, the electric field of a helicon lies in the plane almost normal to the magnetic field. In the general case of $\theta \neq 0$ the polarisation is elliptic. The sign of the polarisation rotation depends on the sign of $(n_e - n_h)/\cos \theta$ and coincides with the direction of rotation of the dominant charge carriers.

The conclusion is that in the long-wavelength limit in metals with an arbitrary closed FS the polarisation of helicons has only a weak dependence on the shape of the FS and is close to that for an isotropic metal (figure 3).
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2.1.3. Closed Fermi surfaces, short waves: $kR \approx 1$—dopplerons. In the vicinity of DSCR (2.33) the spectrum and damping of helicons may change drastically. We emphasise that both the degree of the changes and their character depend strongly on the topology of the FS and the parameters of the electron energy spectrum.

For illustration purposes consider again a metal with a spherical FS. The response of electrons to the electric field of helicons can be determined using the equation of motion:

$$m \dot{v} = e e_0 (v \times B_0) - \frac{m}{\tau} v.$$  (2.41)

The last term in (2.41) describes the dissipation of the electron momentum due to collisions.

Writing for (2.41) the electric field, which propagates along the magnetic field: $e = e_0 \exp [i(kz-\omega t)]$, and taking into account that in view of (2.25) $e_z = 0$, then electrons move along the magnetic field with constant velocity $v_z$, and one obtains

$$\frac{d}{dt} (v_x \pm iv_y) + \left[ -\frac{i\omega e + \frac{1}{\tau}}{m} (v_x \pm iv_y) = \frac{e}{m} (e_{0x} \pm ie_{0y} \exp[i(kv_z - \omega)t]). \right.$$

Solving this equation and averaging velocities in time intervals of the order of the relaxation time and using $\omega \tau >> 1$, the RHS of (2.43) depends on neither $k$ nor $v_z$, and the relationship between the current $j = ne(v)$ and the electric field is identical to that in the static case (2.19).

This is not the case in the vicinity of DSCR. One has to sum contributions into the current of all electrons with different velocities $v_z$. How does this affect the helicon spectrum?

Using (2.20) one can rewrite the dispersion equation (2.22) in the form (Quinn and Rodriguez 1964):

$$c^2 k^2/4\pi \omega = \pm \sigma_x + i\sigma_x = \sigma_z.$$  (2.44)

The quantities $\sigma_z$ can be computed by multiplying (2.43) by $e$ and performing the summation over velocities $v_z$.

Calculations for an arbitrary but axially symmetric FS with the magnetic field oriented along the symmetry axis result in (Chambers 1956)

$$\sigma_z(k, \omega) = \frac{ie^2}{(2\pi)^2} \frac{eB_0}{c} \int_{FS} \frac{d\rho_z}{\omega_c + i/\tau \pm \omega_c - kv_z}.$$  (2.45)

where $\rho_z$ and $v_z$ are the parallel to the field and perpendicular to the field components of the momentum and velocity. After the integral (2.45) is calculated for the spherical FS, equation (2.44) takes the form (Miller and Haering 1962):

$$k^2 = \frac{4\pi ne\omega}{cB_0} \frac{3}{4kR} \left[ (1 - \eta^2) \ln \left( \frac{\eta + 1}{\eta - 1} \right) + 2\eta \right] = \frac{4\pi ne\omega}{cB_0} f(kR)$$

where $\eta = (\omega_c - \omega - i/\tau)/kv_z \approx 1/kR$. 

If \( kR < 1 \), then \( \eta > 1 \) and the argument of the logarithm in (2.46) is positive, so that (2.46) gives real and positive values of \( k^2 \). If, however, \( kR > 1 \), then \( \eta < 1 \) and the argument of the logarithm in (2.46) is negative. As a result \( k^2 \) becomes complex, corresponding to additional attenuation of the wave. The equality \( \eta = 1 \) determines the threshold of DSCR.

As equation (2.46) shows, near the threshold the helicon dispersion equation changes. In the limit of \( kR \rightarrow 1 \) one finds \( f(kR) \rightarrow 3/2 \), so that the phase velocity of the wave decreases (figures 4(c) and (d)). The changes in the helicon spectrum in metals with a spherical FS are relatively weak, since the resonance absorption starts first with electrons which have the maximal value of \( v_z \). However, such electrons have small \( v_\perp \) (figure 4(a)) and their contribution to the non-dissipative component of the conductivity, according to (2.45), is small. The situation changes if electrons with maximal values of \( v_z \) can have large enough transverse velocities.

Consider (Falk et al 1970) the model FS, depicted in figure 4(b). It is reminiscent of the FS of copper, figure 2(a), although some features of the latter, irrelevant for our present purposes, are missing. The model describes the isoenergetic surface for electrons with the spectrum

\[
e = p_\perp^2/2m + (2/\pi)p_0v\sin^2(\pi p_z/2p_0)
\]

where \( p_\perp \) is the component of the momentum perpendicular to the magnetic field and \( p_0 \) and \( v \) are certain parameters with the dimensions of momentum and velocity.
If the Fermi level $\epsilon_f > (2/\pi) v_p p_0$, then the Fermi surface, defined by

$$\epsilon = \epsilon_f = \text{constant}$$

possesses necks at $p_z = (2n + 1)p_0$. Let us take the planes $p_z = \text{constant} = \pm p_0$ for the boundary of the Brillouin zone.

Our model FS possesses axial symmetry and conductivity in the magnetic field along the symmetry axis can be calculated using (2.45). One should recall that for the anisotropic electron spectrum the velocity and the cyclotron frequency are given by equations

$$v = \frac{\partial \epsilon}{\partial p} \tag{2.48}$$

$$\omega_c = \frac{2\pi e B_0}{c} \left( \frac{\partial S}{\partial \epsilon} \right)^{-1}_{p_z} \tag{2.49}$$

where $S(p_z, \epsilon)$ is the area of cross section of the FS by the plane $p_z = \text{constant}$. The velocity $v_z$ acquires its maximal value, equal to $v$, at $p_z = \pm \frac{1}{2} p_0$.

Using the above parameters in (2.45) one obtains (Falk et al 1970):

$$\sigma_{\pm}(k, \omega) = \frac{ie^2 n}{mkv_\eta} (1 - \eta^{-2})^{-1/2}. \tag{2.50}$$

Using this result in the dispersion equation (2.44), one finds for the propagating wave

$$k^2 = \frac{4\pi n e^2 \omega}{c^2 mkv_\eta} (1 - \eta^{-2})^{-1/2}. \tag{2.51}$$

Since $\omega \ll \omega_c$, one can rewrite (2.51) at $\tau \to \infty$ in the form:

$$\omega = \frac{cB_0 k^2}{4\pi ne} \left[ 1 - (k^2 / \omega_c^2) \right]^{1/2}. \tag{2.52}$$

In the long-wavelength limit of $kv \ll \omega_c$ this dispersion relation reduces to (2.23). The spectrum $\omega(k^2)$ deviates substantially from the $k^2$ law and has the end point at $k = \omega_c / v$, which is determined by the position of the DSCR (figure 4(c)). There exist two possible solutions for $k$ at all the values of $\omega$ except $\omega = \omega_m$.

The spectrum of helicons $\omega(k)$ with this two-fold ambiguity included is shown in figure 4(d). Two kinds of electromagnetic waves with positive group velocities but with opposite phase velocity can coexist.

One of these two waves has its origin in DSCR and was called a 'doppleron' (Konstantinov and Skobov 1970). The possible existence of this new kind of weakly damped mode in metals was predicted by Overhauser and Rodriguez (1966) and McGroddy et al (1966). The first unambiguous experimental demonstration of the existence of dopplerons was due to Fisher et al (1971).

2.2. Damping mechanisms

The damping of helicons is usually defined in terms of the ratio of imaginary-to-real components of either the frequency or the wavevector. For weak damping the definitions are connected by

$$\Gamma = \frac{\text{Im} \omega}{\text{Re} \omega} = 2 \frac{\text{Im} k}{\text{Re} k}. \tag{2.53}$$
The first definition characterises the temporal damping of, say, the standing wave in the cavity after the exciting field is switched off, whereas the second one is useful when the wave in the metal is excited by an external electromagnetic field of fixed frequency and determines the spatial attenuation of the helicon amplitude.

Using (2.36) in (2.22), one finds for the damping in a strong magnetic field

$$\Gamma = \frac{\sigma_{xx} \cos^2 \theta + \sigma_{yy} + [\sigma_{yx} \sin \theta + \sigma_{zz} \cos \theta]^2 + \sigma_{zy}^2}{(2|\sigma_{yx}| \cos \theta)}.$$  

(2.54)

2.2.1. Damping in metals with spherical Fermi surfaces. In the local limit of $kl \ll 1$ the use of components of the tensor (2.19) in (2.54) leads for the strong magnetic field, $\omega_c \tau \gg 1$, to the well-known formula:

$$\Gamma = (\omega_c \tau \cos \theta)^{-1}.$$  

(2.55)

Contrary to (2.23), even in this simple case the damping of helicons depends on parameters determined by the energy spectrum of electrons, the structure of the crystal lattice of the metal, temperature and concentration of impurities and imperfections.

Once the cyclotron mass is known, measurements of the helicon damping would determine the relaxation time of the conduction electrons. If the locality condition holds, helicon resonance can be employed to study mechanisms for the scattering of electrons (§ 3.5.2).

If the mean free path increases in the non-local regime, there can appear additional, collisionless damping of helicons. An example of this kind—Doppler-shifted cyclotron resonance—has already been discussed.

There exists one more mechanism of non-local damping—the so-called Landau damping (LD) (Kaner and Skobov 1964). It is present in the long-wavelength regime $kR \ll 1$ and is stronger for shorter helicon wavelengths. The mechanism of LD can most easily be understood in the extreme non-local case $kl \gg 1$.

The motion of electrons in real space can be considered as a superposition of motion with mean velocity $v_0$ along the magnetic field and Larmor circulation with orbital velocity $v_\perp$:

$$v = v_0 + v_\perp.$$  

(2.56)

The coordinate of the electron in real space is

$$r = v_0 t + v_\perp t = v_0 t + R(t)$$  

(2.57)

where $R(t)$ is the instantaneous position of the electron relative to the so-called 'guiding centre', which moves along the magnetic field with mean velocity $v_0$.

Hence the effective field which acts on the electron is

$$e = e_0 \exp [i(kr - \omega t)] = e_0 \exp [i(kv_0 - \omega) t + i k R].$$  

(2.58)

Since $kR \ll 1$, at $kv_0 \tau \approx k_\perp l \gg 1$ the interaction with the wave is most effective for those electrons for which, during the time interval $\tau$, the effective electric field does not vary. This is the case for electrons with

$$kv_0 - \omega = 0$$  

(2.59)

i.e. with velocity along the helicon wavevector equal to the helicon phase velocity:

$$v_0 \frac{k}{\omega} = \frac{\omega}{k}.$$  

(2.60)
Hence the Doppler effect can result in the resonance interaction when the Doppler-shifted frequency (2.32) vanishes.

The phase velocity of helicons is, by many orders of magnitude, smaller than the Fermi velocity, so that the resonance condition holds only for electrons with almost vanishing velocity along the field. For a spherical FS only the near-equator electrons take part in the resonance interaction (figures 4(a) and 5(a) and (b)), whereas polar electrons, near the so-called limiting points, participate in DSCR (figure 4(a)).

The mean energy absorbed by electrons per cyclotron period is (Buchsbaum and Platzman 1967):

$$P = e(eu) = e(e_u)\nu_0 + e(e_u).$$  

(2.61)

Here the first term is the 'conventional' LD due to the longitudinal electric field of the wave. The specific feature of LD for helicons in metals is the practical dominance of the second term in (2.61) due to the transverse components of the electric field of the wave and the orbital motion of electrons in the magnetic field. Indeed, the longitudinal field of the wave, $e_\parallel$, is much smaller than the transverse one (see (2.39) and (2.40)). Moreover, under the resonance condition (2.60) $\nu_0 = \omega/k_\parallel \ll \nu_\perp = \nu_F$. Being determined by the orbital motion of electrons in the magnetic field, the LD of helicons is sometimes called 'magnetic' Landau damping. Let us now consider in more detail the origin of magnetic Landau damping.

The electric field which acts on moving in phase with helicon electrons, while being practically time-independent, is not uniform along the orbit (figure 2(c)):

$$e = e_0 \exp (i k R).$$  

(2.62)

Dropping the small first term on the RHS of (2.61), the mean energy absorbed by an electron per cyclotron period is then

$$P = e(e_u) = ee_0\nu_\perp \exp (i k R).$$  

(2.63)
If the electron orbit in the reference frame co-moving with velocity \( v_0 \) along the magnetic field (guiding-centre frame) is perpendicular to the wavevector \( k \), i.e. \( kR = 0 \), then the magnetic LD vanishes, since

\[
P = e e_0 \langle v_\perp \rangle = 0.
\] (2.64)

Otherwise, even at \( kR \ll 1 \) the LD of the helicons can be substantial and even exceed the collision damping. We emphasise that the origin of the magnetic LD is in small variations of the electric field of the wave along the orbit (see (2.62)). The LD in metals with a spherical FS vanishes for helicons propagating along the magnetic field \( B_0 \) and rises monotonically as the angle between \( k \) and \( B_0 \) is increased since, in the guiding-centre frame, the electron orbits are perpendicular to \( B_0 \): \( R \perp B_0 \) (figures 5(a) and (b)).

Collisions affect the magnitude of LD as follows. On the one hand the number of in-phase electrons is reduced. On the other hand, non-resonant electrons with finite mean free path begin to absorb the energy of the wave between successive collisions (Lampert et al 1966, Buchsbaum and Platzman 1967). The absorption is most effective by electrons in the vicinity of the resonance orbit (2.59), in the belt \( \Delta p_z \) on the FS, with angular dimension of the order of \( 1/k_z l \). At large enough \( kl \), LD does not depend on the mean free path, as the product of the number of electrons contributing to LD and the strength of the interaction with the wave (proportional to the relaxation time) does not depend on it. As a result, in metals with a spherical FS the growth of LD at fixed wavelength saturates at large mean free path of electrons.

The total damping at \( kR \ll 1 \) is a sum of the collision damping \( \Gamma_c \), given by (2.55), and LD, \( \Gamma_L \) (e.g. Halevi and Rabinovich 1970):

\[
\Gamma = \Gamma_c + \Gamma_L = (\omega_c \tau \cos \theta)^{-1} + \frac{\chi(k_z l) \sin^2 \theta}{\omega_c \tau \cos \theta}
\] (2.65)

where

\[
\chi(k_z l) = \frac{3}{8} \frac{(k_z l)^2 \tan^{-1} (k_z l)}{k_z l - \tan^{-1} (k_z l)} - \frac{9}{8}
\] (2.66)

\[
\Gamma_L = \frac{3\pi}{16} kR \sin^2 \theta
\] (2.67)

\[k_z l \to \infty.\]

The factor \( \sin^2 \theta \) in (2.65) and (2.67) is due to the above discussed dependence of the energy absorbed by electrons on the tilt of the orbit relative to the wavevector. The helicon damping in metals with a spherical FS as a function of \( l \) and of the angle between the helicon propagation direction and the magnetic field is depicted in figure 6.

### 2.2.2. Damping in metals with an FS of arbitrary shape.

As was already mentioned above (§ 2.1), the spectrum of helicons is sensitive to the shape of the FS only in the short-wavelength limit, but the damping is FS-topology-dependent both in the short- and long-wavelength regimes.

In the general case the damping helicons \( \Gamma \) can be decomposed into the collision damping \( \Gamma_c \) and non-local damping \( \Gamma_n(k, \omega) \), as was done above for the spherical FS:

\[
\Gamma = \Gamma_c + \Gamma_n(k, \omega).
\] (2.68)
The collision damping (CD) is defined as

$$\Gamma_c = \lim_{k \to 0} \Gamma.$$  \hfill (2.69)

To analyse the damping of helicons further one has to specify the conductivity tensor. Provided that conditions

$$\omega_c \tau \gg 1$$
$$\omega_c \gg \omega$$
$$\omega \tau \ll 1$$

$$k_z v_{0\text{max}} / \omega_c \sim kR \ll 1$$

the conductivity tensor (2.34) for the metal with an arbitrary closed rs in the coordinate system connected with the magnetic field in the long-wavelength limit takes the form (Kaner and Skobov 1968, Halevi and Rabinovich 1970):

$$\sigma_{xx} = \frac{4\pi e^2}{h^3 B_0^2} \sum_{p_{zm}} \int_{-p_{zm}}^{p_{zm}} dp_z \, m_e v_0 \left( \frac{m_e v_0^2}{\tau} + \frac{(k \cdot (v_p)^2)}{\tau - i \omega + i k_z v_0} \right)$$  \hfill (2.71(a))

$$\sigma_{yy} = \frac{4\pi e^2}{h^3 B_0^2} \sum_{-p_{zm}} \int_{p_{zm}}^{p_{zm}} dp_z \, m_e v_0 \left( \frac{m_e v_0^2}{\tau} + \frac{(k \cdot (v_p)^2)}{\tau - i \omega + i k_z v_0} \right)$$  \hfill (2.71(b))

$$\sigma_{zz} = \frac{4\pi e^2}{h^3} \sum_{-p_{zm}} \int_{p_{zm}}^{p_{zm}} dp_z \, \frac{m_e v_0^2}{\tau - i \omega + i k_z v_0}$$  \hfill (2.71(c))

$$\sigma_{xy} = (n_e - n_h) e c / B_0$$  \hfill (2.71(d))

$$\sigma_{xx} = - \frac{(n_e - n_h) e c}{B_0} \tan \theta + \frac{4\pi e c}{h^3 B_0 k_z} \times \sum_{-p_{zm}} \int_{p_{zm}}^{p_{zm}} dp_z \, \frac{m_e k \cdot (v_p)}{(\tau - i \omega + i k_z v_0)^2}$$  \hfill (2.71(e))
The remaining three components can be found using the Onsager relations:

\[ \sigma_{ij}(B_0) = \sigma_{ij}(-B_0). \]  

(2.72)

Here we follow the notations and use the symbols of § 2.1. The brackets \( \langle \rangle \) imply the average over the cyclotron period. The quantity \( P_i \) stands for the oscillating component of the electron’s momentum:

\[ P_i = p_i - \langle p_i \rangle. \]  

(2.73)

The summation in (2.71(a))–(2.71(f)) is performed over all the parts of the Fermi surface.

2.2.2.1. Collision damping. Insert components (2.71(a))–(2.71(f)) into equation (2.54) for the damping. Take the limit of \( k l \to 0 \). According to (2.69) the result for collision damping is

\[
\Gamma_c = \frac{\pi c}{e^2 h^3 |n_e - n_i| B_0 \cos \theta} \left\{ \sum \int_{-p_{zm}}^{p_{zm}} dp_z \tau^{-1} m_e (\langle P_x^2 \rangle + \langle P_y^2 \rangle \cos^2 \theta) \right. \\
+ \left[ \sum \int_{-p_{zm}}^{p_{zm}} dp_z m_e (\langle v_x P_y \rangle \sin \theta + \langle v_z P_y \rangle \cos \theta) \right]^2 \\
+ \left( \sum \int_{-p_{zm}}^{p_{zm}} dp_z m_e (v_x P_y) \right)^2 \left( \sum \int_{-p_{zm}}^{p_{zm}} dp_z m_e \tau^{-1} \right)^{-1} \}.
\]  

(2.74)

In the case of a spherical FS and one kind of carrier one easily gets from this expression the formula (2.55).

In a general case the collision damping (2.74) remains of the order of \( (\omega_c \tau)^{-1} \), where \( \omega_c \) and \( \tau \) stand for the typical cyclotron frequency and relaxation time.

If the collision frequency \( \tau^{-1} \) does not depend on \( p_z \), it becomes the factor in front of the integral and sum, so that collision damping can be written as

\[ \Gamma_c = \tau^{-1} (\Phi(\theta, \varphi)/B_0 \cos \theta) = \tau^{-1} (\omega_c \cos \theta)^{-1}. \]  

(2.75)

Here the function \( \Phi \) depends only on the electron spectrum parameters and the angles \( \theta \) and \( \varphi \) between vectors \( B_0 \) and \( k \) and the lattice axes. The quantity \( \Phi/B_0 \) is typically of the order of \( (\omega_c)^{-1} \).

If the energy spectrum of electrons is sufficiently well known, one can calculate \( \Phi \). Therefore, measurements of the collision damping enable one to determine the collision frequencies of the conduction electrons with other quasi-particles, lattice imperfections or impurities.

Some more comments on the quantity \( \tau^{-1} \) in the formula (2.74) for the damping of helicons are now appropriate. Various processes differ in their contribution to the damping. Evidently, small-angle scattering only weakly shifts the orbit centre along the electric field and gives small contributions to the dissipative current. In terms of the probability, \( w(p, p') \), of transitions from the state with momentum \( p \) to a state with momentum \( p' \), the dissipative current is (see, for example, Gantmakher and
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Petrashov (1976)

\[ j_D \sim \frac{\tau_{\text{eff}}}{B_0^2} e \sim B_0^2 \sum_{\bar{p}} (1 - \cos \psi) w(\bar{p}, \bar{p}') e. \]  

Here the averaging is over the Fermi surface. Obviously, the dissipative current (2.76) vanishes if there is no scattering: \( w(\bar{p}, \bar{p}') = 0 \). The factor \( (1 - \cos \psi) \), where \( \psi \) is the scattering angle, is a manifestation of vanishing dissipation for the small-angle scattering.

The same factor \( (1 - \cos \psi) \) persists in the conductivity in zero magnetic field (2.15), since again only the large-angle scattering contribution is effective. However, in that case one has to compute the inverse of the quantity in (2.76), as the maximal contribution to the electric current comes from electrons with minimal scattering probability, whereas in the magnetic field the larger the probability of scattering the larger is the contribution to the dissipative current.

The relaxation time with the factor \( (1 - \cos \psi) \) included is called the transport relaxation time. According to the above discussion the relaxation times for zero and strong magnetic fields may differ. One has to bear this in mind when the scattering is anisotropic, i.e. when \( w(\bar{p}, \bar{p}') \) is a steeply varying function of the position on the Fermi surface.

The case of the scattering of electrons on phonons at low temperatures is a peculiar one, since while computing the conductivity tensor one has to maintain the balance of the quasi-momentum in the electron-phonon system (Gurzhi and Kopeliovich 1974, 1981) and to take into account properly the strong anisotropy of the electron-phonon scattering (Pippard 1968, Young 1968, Gantmakher 1974).

2.2.2.2. Landau damping. If \( k_z l \approx 1 \), the dissipative contributions (2.71(a))-(2.71(f)) to the damping, which have a resonance behaviour, should be taken into account. They describe the Landau damping. Its origin has already been demonstrated for a metal with a spherical FS. Some comments and peculiarities of LD in metals with an anisotropic FS are in order.

One of these is due to the velocity of the electron \( v_z \) in the guiding-centre system (2.56) and when, consequently, the orbit is not necessarily perpendicular to the magnetic field, if the FS is anisotropic. This can result in a substantial LD of helicons propagating along the magnetic field, whereas in metals with a spherical FS there is no LD in such a geometry\(^\dagger\). Furthermore, in metals with a non-spherical FS the LD may vanish altogether even at finite angles of propagation with respect to the magnetic field.

The directions of the magnetic field at which LD vanishes or has its local minimum can be found following the geometric procedure of figure 5. It is based on the observation that the electron’s velocity is always normal to the FS and that only electrons with small mean velocities along the wavevector contribute to the LD. Orbits which lie in a plane of constant phase do not contribute to LD (Halevi et al 1969).

\(^\dagger\) The physical picture of the Landau damping put forward by Buchsbaum and Platzman (1967), and quoted in the review by Maxfield (1969), apparently does not give an adequate description of the phenomenon. Indeed, it is based on the confinement of electrons by mirror fields, created via a modulation of the uniform constant magnetic field \( B_0 \) by the magnetic field of the helicon. However, in the case of helicon propagation along the magnetic field in a metal with tilted electron orbits with respect to the magnetic field, the mirror fields do not exist, whereas the Landau damping does. This was first noticed by Skobov (1971, see Maxfield (1971)).
The second important difference of LD for the anisotropic FS from that for the isotropic one is that LD may vanish or have a minimum at several orientations of the magnetic field as is shown in figure 5(e) for the model FS. The possible existence of a few minima of LD was first confirmed experimentally by Krylov (1968a, b).

The dependence of LD on the mean free path in metals with the anisotropic FS is not described by a universal function like (2.65) and may qualitatively differ from that for metals with a spherical FS. For instance, it may well happen that LD vanishes both in the local limit of small mean free paths, \( kl \ll 1 \), and in the extremely non-local limit of large mean free paths, \( kl \gg 1 \), whereas at medium mean free paths it may be substantial and, in principle, much larger than the collision damping. Such a non-monotonic \( kl \) dependence of LD can take place, for example, in metals with non-ellipsoidal FS, such that the orbit tilts relative to the magnetic field and, consequently, the wavevector depends on the position of the orbit on the FS (Volskii and Petrashov 1973). For the model FS of figure 5(e), if the magnetic field is directed along directions I, II and III, the orbits of resonance electrons are perpendicular to the wavevector and those electrons do not contribute to LD. Hence at \( kl \approx 1 \) only non-resonant electrons with orbits not perpendicular to the wavevector can contribute to LD.

One can estimate quantitatively the LD for an anisotropic metal by substituting the resonance terms in (2.71(a))–(2.71(f)) into the damping (2.54):

\[
\Gamma_L \sim \text{Re} \frac{1}{B_0} \int_{-p_z}^{p_z} dp_z \frac{k^2 |G|^2}{1 + ikcl}\Delta
\]

where \( \Delta = \frac{v_z}{v_F} - \omega / k_z v_F \) is the relative deviation of the electron’s mean velocity from the resonant one and \( G^2 = (k/k)(v/v)P_l \).

Irrespective of the form of the functions in (2.77)

\[
\Gamma_L \rightarrow 0 \quad \text{when} \quad kl \rightarrow 0.
\]

This is the local limit and LD vanishes. In the extreme non-local case

\[
\Gamma_L \rightarrow kG^2 (p_z^0)/B_0 \quad \text{when} \quad kl \rightarrow \infty
\]

where \( p_z^0 \) is a value of \( p_z \) at which \( \Delta(p_z) = 0 \). The angular dependence of LD and the dependence on the mean free path is controlled by the form of the functions \( G \) and \( \Delta \). When the scattering of electrons is anisotropic, allowance should be made for the possible dependence of the mean free path \( l \) in (2.77) on \( p_z \).

The LD may lead to a strong smearing of the DSCR edge, thus causing obstacles in employing the edge for determination of FS parameters as proposed earlier (e.g. Maxfield 1969).

### 2.2.2.3. The influence of open orbits.

If the magnetic field is normal to the open direction of the FS (figure 2(a)), then a fraction of the electrons will be in open orbits, thus strongly affecting the conductivity of the metal in a strong magnetic field.

Contributions of different groups of electrons to the conductivity are additive. Hence one can write the total conductivity \( \hat{\sigma} \), at least in the local limit, as

\[
\hat{\sigma}_t = \hat{\sigma}_c + \sum_i \hat{\sigma}_{op}.
\]

Here the sum is over all the open trajectories and \( \hat{\sigma}_c \) is the conductivity due to the closed trajectories and computed using the counterparts of (2.71(a))–(2.71(f)).

For an illustration consider the case of the magnetic field normal to the mirror plane of the crystal when the current due to electrons on open trajectories has no
component along the magnetic field. In this case (Hui 1969):

$$\hat{\sigma}_{op} = \frac{n_{op} e^2 \tau}{m} \begin{vmatrix} \cos^2 \varphi & \cos \varphi \sin \varphi & 0 \\ -\sin \varphi \cos \varphi & \sin^2 \varphi & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

(2.81)

where $\varphi$ is the angle between the normal to the opening and the $x$ axis in real space.

How do the open orbits affect the propagation of helicons? Let the closed part of the FS be spherical and $\varphi = 0$ and $n_{op} \ll n$ (figure 2(a)). Then at $\omega_c \tau \gg 1$:

$$\hat{\sigma}_r = \begin{vmatrix} \sigma_0 & n_{op} e^2 \tau & \mp \frac{\sigma_0}{\omega_c} & 0 \\ \frac{n_{op} e^2 \tau}{m} & \frac{\sigma_0}{\omega_c} & 0 & 0 \\ \mp \frac{\sigma_0}{\omega_c} & \frac{(\omega_c \tau)^2}{m} & 0 & 0 \\ 0 & 0 & \frac{\sigma_0}{\omega_c} & 0 \end{vmatrix}$$

(2.82)

For helicons propagating along the magnetic field the damping is

$$\Gamma = \frac{1}{\omega_c} + \frac{n_{op}}{2n} \frac{\sigma_0}{\omega_c \tau}.$$ 

(2.83)

We see that the presence of even a very small fraction of open orbits, $n_{op}/n \sim (\omega_c \tau)^{-2}$, results in a contribution to the damping comparable with that of the whole FS, whereas the spectrum of helicons does not appreciably change at $n_{op}/n \ll (\omega_c \tau)^{-1}$. The damping increases steeply at strong magnetic fields, so that above a certain field $B_0 > B^*$ propagation becomes impossible. Here

$$B^* \approx \frac{2n mc}{n_{op} e} \tau^{-1}.$$ 

(2.84)

The spectrum of helicons changes as well at fields close to $B^*$ and with a large enough fraction of open orbits (Kittel 1963, Bass et al 1963, Buchsbaum and Wolff 1965).

2.2.2.4. The influence of the magnetic breakdown. One of the reasons why electrons in metals in a strong magnetic field move on complex trajectories (see, for example, figure 1(b)) is the Bragg scattering of electrons by the periodic potential of the crystal lattice. Starting with orbit a of figure 2(d), for certain directions of momentum the Bragg scattering induces a transition into trajectory c with probability $W$ and with probability $P = 1 - W$ into classical trajectory b. A number of phenomena, the relativistic spin–orbit coupling being the most renowned one (see, for example, Pippard (1969)), may forbid the transition from the segment of a into the segment of b altogether, so that the electron can move only along the trajectory a–c (figure 2(d)).

Cohen and Falicov (1961) have discovered that the magnetic field can affect the transition probabilities and give rise to a finite magnetic-field-dependent probability of transition from orbits of type a–c to orbits of type a–b (see figure 2(d)). The phenomenon is called magnetic breakdown.

In a strong enough magnetic field the breakdown probability $P$ may become of the order of unity, so that only the orbit a–b becomes allowed. Hence the magnetic field can radically change the form of the electron orbits, causing conversion of closed orbits into open ones (figure 2(d)) and vice versa (for details see, for example, Cracknell and Wong (1973)), and lead to changes in the components of the conductivity tensor and to the helicon properties.
2.3. Helicon resonators

To study helicons experimentally one excites them in finite-size metallic samples placed in a strong magnetic field by applying an oscillating weak electromagnetic field, by means of an excitation coil. To calculate the response of the sample to the oscillating external field one has to solve the electromagnetic boundary value problem. As the magnetic field and the Hall current are present, the variables in the wave equation separate only in the Cartesian and/or cylindrical coordinate systems. Hence, whilst solving the boundary value problem, one encounters clashing symmetries of the helicon wavefunction, crystal lattice and sample shape. For this reason complex cases were considered only after the simpler ones had been solved.

The first exact solution was obtained for the plane-parallel infinite plate of metal with a spherical FS (Chambers and Jones 1962, Cotti et al 1963, Legendy 1964). Next came the solution of the boundary value problem for the infinite plane-parallel plate of a metal with an anisotropic FS (Bass et al 1963, Penz 1967). Legendy (1964) and Klozenberg et al (1965) have solved the boundary value problem for the infinite cylinder oriented along the magnetic field, and Garland and Bowers (1969) have produced an approximate solution for a cylinder arranged perpendicular to the magnetic field.

For a long time there were no exact solutions for the finite-size samples. The finite-size corrections to the exact solutions for the plate and the cylinder oriented along the magnetic field were calculated by Chambers and Jones (1962), Legendy (1964) and Harding and Thonemann (1965). Those approximate solutions, for instance for the plate, describe the experimental data very well (Amundsen 1966, Amundsen and Jestard 1972).

Ford and Werner (1973) have solved exactly the boundary value problem for a spherical sample of a metal having a spherical FS and isotropic relaxation time, described by the conductivity tensor (2.19). This is the first exact solution for a finite-size sample. In the numerical analysis the solution to the wave equation is expressed as an infinite series of vector spherical harmonics, which comply with the boundary conditions.

Simultaneously the same problem was treated by Fezer (1973) using an expansion in terms of cylindrical waves. He searched for a solution obeying the approximate boundary conditions on the sphere. Although approximate, the resulting solution is of simpler physical structure to that of Ford and Werner (1973).

As yet, no exact solutions exist for finite-size samples of metals with an anisotropic FS.

2.3.1. Helicon resonance in plates. There exist two types of solutions of the boundary value problem for the plane-parallel plate geometry.

The first one corresponds to the experimental geometry of figures 7(a) and (b): the exciting magnetic field is unidirectional on each side of the sample, i.e. symmetric-magnetic-field excitation. The second one corresponds to the asymmetric-magnetic-field excitation when the magnetic field on the two sides of the plate is of opposite phase.

One can easily solve the boundary value problem for the infinite plane-parallel plate, since in the Cartesian coordinate system the variables separate and one needs only a few plane waves to satisfy the boundary condition of continuity of magnetic field at the metal–vacuum boundary (in this geometry the magnetic field and induction are equal).
2.3.1.1. Symmetric excitation. Let the plate lie on the OX'Y' plane and the OZ' axis be normal to the plate. Consider the exciting magnetic field directed along OX' so that \( h_0 = (h_0 \exp(i\omega t), 0, 0) \). Let the strong external field be in the plane OY'Z' with an arbitrary angle relative to the OZ' axis (figure 3(b)). The boundary condition reads \( h|_{z=d/2} = h_0 \), \( d \) being the plate thickness. It is satisfied by a superposition of four circularly polarised waves having the dispersion law (2.22):

\[
h = \frac{h_0 \exp(i\omega t)}{f_+ - f_-} \left( f_+ \cos k_z z - f_- \cos k_z z \right) \left( \cos \frac{1}{2} k_z d \cos \frac{1}{2} k_z d' \cos \frac{1}{2} k_z d \cos \frac{1}{2} k_z d' \right)
\]

\[
f_z = h_x / h_y \quad |z| \leq d/2.
\] (2.85)

One easily finds that for resonance

\[
k(\omega, B_0) = \frac{(2n-1)\pi}{d} \quad (n = 1, 2, 3, \ldots).
\] (2.86)

Thus when resonance occurs, the sample thickness is equal to an odd number of half a wavelength of the helicon.

2.3.1.2. Resonance lineshape under symmetric excitation. A conventional technique is to measure the signal induced in a pick-up coil wound onto the sample. In the method of crossed coils, when the excitation and pick-up coil planes are perpendicular (figure 7(a)), the measured signal is

\[
V_T = i\omega G_\mu_T J_1
\] (2.87)

where \( J_1 \) is the current in the excitation coil, \( G \) is a geometrical factor and \( \mu_T \) stands for transverse permeability:

\[
\mu_T = \frac{h_x}{h_0}
\] (2.88)

where one averages over the sample thickness. Since the exciting field lies in the plane of the pick-up coil, the signal vanishes if there are no helicons.

In the method of parallel coils (figure 7(b)), the measured signal is

\[
V_L = i\omega G_\mu_L J_1
\] (2.89)

where \( \mu_L \) stands for longitudinal permeability:

\[
\mu_L = \frac{h_x}{h_0}.
\] (2.88')

One can expand permeabilities in terms of the resonance contributions (Penz 1967):

\[
\mu_T = -\frac{8}{\pi^2} A_T \sum_{n=1}^{\infty} \frac{(2n-1)^{-2}}{1 + iQ[(\omega/\omega_n) - (\omega_n/\omega)]}
\]

\[
\mu_L = 1 - \frac{8}{\pi^2} A_L \sum_{n=1}^{\infty} \frac{[A_L + iQ(\omega/\omega_n)]}{1 + iQ[(\omega/\omega_n) - (\omega_n/\omega)]}
\] (2.90)

where

\[
\omega_n = [(2n-1)^2 \pi c^2 / 4d^2](\rho_{xx} \rho_{yy} + \rho_{xy} \rho_{yx})^{1/2}
\]

\[
Q = (\rho_{xx} \rho_{yy} - \rho_{xy} \rho_{yx})^{1/2}(\rho_{xx} + \rho_{yy})^{-1}
\] (2.91)

\[
A_T = \rho_{xy} (\rho_{xx} + \rho_{yy})^{-1}
\]

\[
A_L = \rho_{xx} (\rho_{xx} + \rho_{yy})^{-1}.
\]
Here the resistivity tensor $\rho_{ik}$ is calculated in the coordinate system $OX'Y'Z'$ connected with the plate, i.e. in essence the helicon wavevector, see figure 3(b).

To make the connection between the resonance parameters and the microscopic properties of the metal more explicit, we rewrite (2.91) in terms of components of the conductivity tensor (2.71(a))-(2.71(f)) in the coordinate system $OXY2$ connected with the magnetic field $B_0$ (see figure 3(a)). We recall that $Q$ and $\omega_n$ are invariant quantities independent of the coordinate system. In a strong magnetic field:

$$\omega_n = [(2n - 1) \pi / d]^2 (c^2 \cos \theta / 4 \pi |\sigma_{xx}|)$$

$$Q = (2\Gamma)^{-1} = A_T = \left( \begin{array}{c} \sigma_{xx} \cos^2 \theta + \sigma_{xy} + \left[ (\sigma_{yx} \sin \theta + \sigma_{zz} \cos \theta)^2 + \sigma_{zy}^2 \right] / \sigma_{zz} \end{array} \right)^{-1}.$$  \hspace{1cm} (2.92)

Here $\theta$ is an angle between the magnetic field and the normal to the plane of the plate.

One can conclude from the above formulae that by measuring parameters of the helicon resonance in plane-parallel plates one can determine many components of the conductivity and/or resistivity tensor of the metal in a strong magnetic field. Such a possibility was first demonstrated by Chambers and Jones (1962).

2.3.1.3. Free helicon oscillations. Abrupt switching-off of the direct current in the excitation coil is one of the methods of exciting free helicon oscillations in the sample.
The measurements of the spectrum and damping of these oscillations would provide the same information as the technique of forced oscillations. Bowers et al (1961) were the first to use the former method to discover helicons.

If a weak permanent field $h_{0x}$ is switched off at $t=0$, then for $t>0$ the time dependence of the magnetic field in the plate of an isotropic metal is given by (Chambers and Jones 1962)

$$h(t)/h_{0x} = 4\pi^{-1} \sum_{n=1}^{\infty} (2n-1)^{-1}(2n-2) \cos[(2n-1)\pi z/d] \times \exp[-(i\omega_n + 1/\tau_n)t]. \quad (2.93)$$

Here $h(t) = h_x + ih_y$. When considering the time dependence of free oscillations, one takes the wavenumber in (2.22) to be real and the frequency to be complex. The values of $\omega_n$ in (2.93) are given by formulae (2.91) and (2.92): $1/\tau_n = \omega_n/\omega_c \cos \theta$ in accordance with (2.53) and (2.55). The mean magnetic field, averaged over the sample thickness, which affects the pick-up coil, is given by a superposition of exponents:

$$\overline{h(t)/h_{0x}} = 8\pi^{-2} \sum_{n=1}^{\infty} (2n-1)^{-2} \exp[-(i\omega_n + 1/\tau_n)t]. \quad (2.94)$$

Oscillations corresponding to the higher helicon modes, $n>1$, fade much faster, so that after the time $\tau_1$ of fading of the fundamental helicon mode the contribution of higher modes is negligibly small.

All the above formulae hold also to a high accuracy for metals with an arbitrary closed FS if the damping of helicons is sufficiently weak. Then the distribution of magnetic field in the sample and its polarisation do not differ markedly from those for the isotropic case, as was already discussed in §2.1.2. The decay time of the $n$th helicon mode is related to the frequency and $Q$ factor in formulae (2.91) and (2.92) by

$$1/\tau_n = \omega_n/2Q. \quad (2.95)$$

### 2.3.1.4. On the finite-size corrections.

The resonance frequencies $\omega_{nlm}$ for a parallelepiped of dimensions $a \times a \times d$ are given by the formula, proposed by Legendy (1964):

$$\omega_{nlm} = \omega_n F_{nlm}. \quad (2.96)$$

Here $n, l, m$ are integers. For the fundamental resonance at $\eta = a/d \leq 0.25$ one finds $F_{111}(\eta) = 1 + 2\sqrt{2}\eta/\pi$ (Amundsen 1966). The resonance curves in Legendy's theory are, in general, broader than those given by formulae (2.91) and (2.92). However, this broadening is negligible at $(\omega_c \tau)^{-1} \ll 1$ (Amundsen and Seeberg 1969). Legendy's formula (2.96) agrees with the data on the position of the resonance peaks as a function of $\eta$ (Amundsen 1966, Amundsen and Seeberg 1969, Amundsen and Jestard 1972) better than the alternative formulae proposed by other authors (e.g. Merrill et al 1963, Cotti et al 1963).

### 2.3.2. Helicon resonance in a sphere.

The use of spherical samples is particularly advantageous, as using a single sample one can study all the parameters of helicon resonance as a function of magnetic field and temperature at all orientations of the magnetic field relative to the crystal lattice axes. This is not possible with plates since at small enough angles between the magnetic field and the plane of the plates the...
helicons cannot be observed. One is therefore forced to use different samples having
different crystallographic orientations. Additional uncertainties are thereby intro-
duced, particularly if the parameter of interest, such as damping, depends strongly on
sample preparation. Spherical samples are also advantageous in studies of helicon
resonance in ferromagnetics, since the magnetisation of a sphere is readily calculated.

There are two customary techniques for the excitation of helicon resonance in a
sphere.

(a) The transverse geometry in which both excitation and pick-up coils are arranged
perpendicular to the strong uniform field (figure 7(c)).

(b) The longitudinal geometry with the excitation coil axis parallel to the magnetic
field (figure 7(d)), first proposed by Ford and Werner (1973), Werner and Ford (1975)
and also by Werner et al (1974).

2.3.2.1. Transverse geometry of excitation. In this case the signal in the secondary coils
is described by formulae analogous to (2.87)–(2.92) for resonances in the plate, though
the structure of the resonances is much more complicated.

There are two classification systems for the resonances in a sphere. Ford and
Werner (1973) have proposed to label resonances by a pair of integers \((m, n)\), where
\(m\) and \(n\) stand for the number of nodes of the magnetic field of the wave along the
OZ and OX axes, respectively, with an external field directed along the OZ axis.
Fezer (1973) proposed to classify resonances as one does in plates: by the number of
'half-waves' in the sample along the magnetic field (OZ axis) and in the radial direction
in the OXY plane.

The resonance 'frequency' \(V\), amplitude \(A\) and \(Q\) factor of the fundamental
resonance, \((1, 0)\) in the Ford and Werner (1973) classification or \((1, 1)\) in Fezer's
classification, have the following dependence on the magnetic field and sample size
according to the numerical calculations of Werner and Ford (1975):

\[
V_0 = 4.9020 W(1 + 2.90/ W^2) \\
A = 0.2780 W(1 + 6.12/ W^2) \\
Q = 0.2349 W(1 - 2.64/ W^2)
\] (2.97)

where the two dimensionless quantities

\[
V = 4 \pi \omega \sigma_0 a^2/ c^2 \\
W = \omega_c \tau
\]
can be regarded as a scaled frequency and a scaled magnetic field, respectively. At
\(W > 7.5\) these formulae are accurate to within 0.3\%. The quantity \(A\) is in units of
\(\frac{1}{4} h_0 a^3\), where \(h_0\) is the excitation field amplitude and \(a\) is the sphere radius.

2.3.2.2. Longitudinal geometry of excitation. Under the longitudinal excitation of
helicons in a sphere resonance occurs in the component of the sphere's magnetic-dipole
moment \(M_z\) parallel to the magnetic field. For the fundamental longitudinal resonance
the numerical calculations give (Werner and Ford 1975):

\[
V_0 = 21.78 W(1 + 3.83/ W^2) \\
Q = 0.2920 W(1 - 81.5/ W^2) \\
A = 0.02315 W(1 + 76.9/ W^2)
\] (2.98)

using the notation of (2.97). At \(W > 15\) these formulae are accurate to within 0.4\%.
Measurements in the longitudinal geometry have certain advantages, as the torque due to the interaction of the magnetic moment of the sample with the external constant magnetic field, which may distort the results of measurements of parameters of helicon resonance (see § 3.2), is absent.

2.3.3. On resonances in samples of arbitrary shape. Apart from the above examples, much work has been done on resonances in cylinders with their axes aligned parallel to the magnetic field (Klozenberg et al 1965, Legendy 1964). The solution of the boundary value problem for helicons in such a sample does exist, since in the cylinder system the variables do separate. However, a practical use of cylindrical samples is limited by the so-called surface losses (Legendy 1964), as there is a large surface area parallel to the magnetic field, which cannot easily be taken into account when measuring the helicon damping.

Quite often one uses cylindrical samples with their axes arranged perpendicular to the magnetic field. This geometry was used by Bowers et al (1961) in their discovery of helicons in sodium (see figure 7(e)). Despite the absence of an exact solution of this boundary value problem (see, however, an approximate solution by Garland and Bowers (1969)), one can still formulate a few properties of helicon resonance which hold for arbitrary shaped samples. They are implied by the wave equation and the dispersion law (2.38).

In the strong magnetic-field limit, helicon resonances satisfy

$$\omega |\sigma_{yx}| = \text{constant.} \quad (2.99)$$

In the uncompensated metal with a closed FS, $\sigma_{yx} \sim (n_e - n_h)/B_0$ (see (2.71)) so that the resonance frequency is proportional to the magnetic field, inversely proportional to the charge carrier concentration and, in view of the quadratic behaviour of the spectrum, to the square of the sample size $D$:

$$\omega = \text{constant} \times B_0 (|n_e - n_h| D^2)^{-1}. \quad (2.100)$$

If the sample has surfaces with small enough area parallel to the magnetic field and the surface losses are small, then in the strong field and local limit the $Q$ factor is of the order of the product of the typical cyclotron frequency and relaxation time.

We have discussed above the solutions of the boundary value problem for the symmetric magnetic-field excitation of resonances. Most of the results to be discussed below were obtained in this geometry. An analysis of the asymmetric excitation can be found in reviews by Kaner and Skobov (1968) and Maxfield (1969).

3. Experimental study of helicon resonance in the absence of quantum effects

3.1. Methods of experimental observation of the resonance

The resonance condition (2.86) is represented in figure 7(f) by horizontal lines, $k = k_{res} = \text{constant}$. As the magnetic field $B_0$ and frequency $\omega$ are varied, one moves along the curve $k(\omega, B_0)$. Resonances correspond to the intersection of this curve with lines $k = k_{res}$. Curves corresponding to the most common types of experiments are indicated.

(a) Helicon resonances are recorded by varying $B_0$ at a fixed excitation frequency. Experiments of this type are advantageous for low signal-to-noise ratios, as one can use narrow-band detection. In plates one observes a sequence of resonances at
magnetic fields

\[ B_n = (2n-1)^{-2}4|n_e - n_h|eωd^2/πc \cos \theta. \]  

(3.1)

The higher-order resonances are observed at weaker fields compared to the fundamental one with \( n = 1 \).

(b) Resonance is observed by varying the frequency at constant \( B_0 \). This technique is convenient when helicon parameters depend strongly on the magnetic field and vary appreciably over the linewidth \( ΔB = B_0/ω_cτ \). This is a case, for instance, under the conditions of magnetic breakdown and, particularly, in the presence of quantum effects (§§ 4 and 5). Resonances are observed at frequencies given by formulae (2.91) and (2.92); the fundamental resonance corresponds to the minimal frequency.

Some commonly used circuits are shown in figures 7(g) and (h). One can also use circuits based on the Pound generator (e.g. Krylov 1968a, b) or a conventional bridge.

(c) As first proposed by Houck and Bowers (1964), resonances are observed for the condition \( k = k_{res} = \) constant. A practical circuit is shown in figure 7(i). One takes two crossed coils. The signal from the pick-up coil is amplified, shifted in phase by \( π/2 \) and fed back to the secondary coil, so that the sample is a part of the feedback circuit, causing the system to become naturally self-oscillatory at the fundamental frequency of helicon resonance. Houck and Bowers (1964) have proposed that such a 'helicon generator' is useful for measurements of the intensity of magnetic fields at low temperatures. Later on Volskii and Petrashov (1968, 1970) proposed the use of a helicon generator for absolute measurements of the amplitude of the de Haas–van Alphen effect (§ 5). Lausmaa et al (1983) have used a circuit in which the self-oscillations may be locked to any mode of the helicon resonance (see § 3.2).

(d) Free helicon oscillations are registered and recorded directly as a function of time (Bowers et al 1961, Krylov et al 1974, Krylov 1976).

Typical signals employing these methods are shown in figures 8 and 17(d). In all cases the drive coil consists of 10–100 turns with an excitation current of 1–100 mA.

**Figure 8.** Typical experimental records of helicon resonances in indium. (a) In a plate, geometry of figure 7(a), \( ω = \) constant curves. In plate, \( d = 0.7 \) mm, \( T = 1.3 \) K. 1, 502 Hz; 2, 3.6 kHz; 3, 13.3 kHz. (b) In a spherical sample, geometry of figure 7(c), \( ω = \) constant. In sphere, diameter 10 mm. 1, 3.3 Hz; 2, 12.3 Hz, \( T = 1.3 \) K. (c) Free oscillations in a cylinder, geometry of figure 7(e). In cylinder, \( d = 4 \) mm. 1, intermediate state (\( H = 185 \) Oe); 2, normal state (\( H = 245 \) Oe); \( T = 1.23 \) K.
The pick-up coils contained 100–1000 turns yielding a resonance signal of typically 100 mV.

At frequencies exceeding 100 MHz one usually registers the signal transmitted through the sample (Grimes and Buchsbaum 1964, Libchaber and Grimes 1969). The relevant signals are described by the asymmetric-field solutions of the boundary value problem (Bass et al 1963, Penz 1967).

We conclude this subsection by citing metals in which helicons have been observed: Na (Bowers et al 1961); Li, K, Al, In (Chambers and Jones 1962); Cu (Cotti et al 1963); Ag, Au, Sn, Zn, Cd (Taylor et al 1963a, b); Ni (Grimes 1964); Pb (Grimes et al 1965); Nb (Druyvestyn et al 1966); Tl (Yamamoto 1981).

3.2. Acoustic satellites of helicon resonance

Apart from the resonances described by formulae (2.90)–(2.96), there may exist additional satellites, first discovered in studies of helicon resonances in plane-parallel single-crystal plates of very high purity indium (Petrashov 1979). The experimentally observed peak structure is presented in figure 9(a). In a weak magnetic field it is a simple one, corresponding to the fundamental resonance with \( n = 1 \) in equation (2.94).

Figure 9. Acoustic satellites of helicon resonance, experimental records. (a), (b) Generation of satellites recorded at (a) different fixed frequencies (1, 22.1 Hz; 2, 66.3 Hz; 3, 110.5 Hz) and (b) different fixed magnetic fields, (c) shows helicon resonance with \( n = 2 \). Note that satellites are absent. Experiments refer to an indium crystal plate as dimensions 1.5 \( \times \) 10 \( \times \) 50 mm\(^3\). \( T = 1.3 \) K. Left: \( f = 132.6 \) Hz; right: \( f = 1190 \) Hz.
The Hall constant, deduced from the peak position, is $1.6 \times 10^{-12} \, \Omega \, \text{cm} \, \text{G}^{-1}$ and agrees to within 1% with alternative determinations (Harding and Thonemann 1965, Amundsen 1966).

As the field strength is increased, the peak structure becomes more complex. The shape of the peak is first distorted and this is followed by the peak splitting into a series of resonance peaks. The phenomenon is observed both with fixed-frequency and fixed magnetic-field approaches (figures 9(a) and (b)). An analysis of the peak positions shows that the linear dependence of the resonance magnetic field on the excitation frequency and vice versa, typical for helicons, persists only for peaks $\beta$, whereas for peaks $\alpha$, observed in weaker fields, and peaks $\gamma$, observed in stronger fields, it is non-linear (figure 10(c)).

In finite-sized plates, when the resonance conditions involve all three dimensions, satellites may also exist. However, these are of conventional helicon origin, and should

![Diagram](image-url)

**Figure 10.** (a) Flexural oscillations of a metallic plate in a magnetic field, (b) velocity of the acoustic flexural waves in a metallic plate. The dotted curve is for zero magnetic field. The position of the helicon resonance is shown by a vertical chain line; (c) positions of resonances observed in a plate plotted against frequency.
only appear either in the high-frequency tail, if one keeps $B_0 = \text{constant}$, or at lower magnetic fields if $\omega = \text{constant}$. Like the principal helicon resonances these satellite resonances display a proportionality of resonance frequency to the magnetic field (see equations (2.92) and (2.96)). Thus, the results of figures 9(a) and (b) are at variance with the theory of 'classical' helicons at least in two respects: firstly, the proportionality of the resonance field to the excitation frequency breaks down; and secondly, there exist satellites at frequencies below the helicon resonance frequency (peak $\gamma$). These anomalous satellites disappear once higher-order helicon resonances, with $n = 2, 3, \ldots$, are excited and are extremely sensitive to the manner of fixing the sample and to its orientation with respect to the set of coils.

An analysis of the results has demonstrated that the observed satellites are connected with flexural acoustic waves, the spectrum of which is strongly influenced by the magnetic field. This can be understood as follows. Consider a metallic plate with its surface normal arranged parallel to a strong magnetic field (figure 10(a)). Any flexure and/or rotation of the plate about an axis not parallel to the magnetic field induces a tangential component of the magnetic field, which can excite helicons, in the absence of an excitation coil, by bending or rocking the sample with frequency $\omega_v$.

In turn, excitation of helicons may result in a resonance-enhanced magnetic moment of the plate, which can interact with the external magnetic field, causing the rocking or bending of the plate. Such effects which change the amplitude of helicon resonance have been described by Maxfield (1969) and were first studied quantitatively by Delaney and Pippard (1971, 1972). A consistent solution of the problem of acoustic vibrations of a plate with allowance for helicon propagation, which allows one to understand the origin of additional resonances and their basic properties, is given by Petrashov (1979).

One starts by calculating the linear magnetisation $m$, averaged across the plate, with boundary conditions implied by the acoustic vibrations:

$$h(x, t)|_{z = \pm d/2} = B_0 \theta(x, t).$$  \hspace{1cm} (3.2)

Here $h$ is the tangential component of the magnetic field and $\theta$ is the angle between the external field and the normal to the plate (figure 10(a)). Then one calculates the bending moment (torque) density $K = (m \times B_0)$, which enters into equations for the flexural acoustical vibrations of the plate. The resulting dispersion equation for the flexural vibrations of the plate with helicons propagating, oriented perpendicular to the magnetic field $B_0$, is

$$q^2 = \frac{\mathcal{K}B_0^2 \pm (\mathcal{K}^2 B_0^4 + \frac{1}{3} \rho E d^2 \omega^2)^{1/2}}{\frac{1}{4} E d^2}$$  \hspace{1cm} (3.3)

where

$$\mathcal{K} = \frac{1}{4\pi} \left\{ \tan \left( kd/2 \right) \left( kd/2 \right)^{-1} - 1 \right\}$$  \hspace{1cm} (3.4)

$q$ and $k$ are the wavenumbers of the acoustic and helicon waves, $\rho$ is the metal density and $E$ is the Young's modulus. The '+' sign in (3.3) corresponds to the acoustic wave, modified in the presence of helicons, and propagating along the plate across the magnetic field (figure 10(a)). At $\mathcal{K} = 0$ the dispersion equation coincides with the familiar one for zero magnetic field (Skudrzyk 1968).

As seen from (3.3) and (3.4), in the vicinity of the helicon resonance the acoustic wave parameters have steep variations. As all the quantities in (3.3) and (3.4) are well known, one can easily perform numerical calculations for the cases of interest.
The typical variations of the velocity of the flexural acoustic waves $c_s$ in the indium plate, calculated for particular plate parameters, are shown in figure 10(b). The damping of helicons is neglected. The effective elasticity of the plate is decreased at lower and increased at higher frequencies. This is due to the fact that at higher frequencies the elastic and magnetic torques are parallel to each other, whereas at lower frequencies they are antiparallel.

In the helicon resonance region the wavelength of the acoustic vibrations is a steep function of both frequency and magnetic field. The experimentally observed satellites correspond to a given number of acoustic wavelengths in the transverse dimension $L$ of the plate. The other dimension was too small for resonances to occur.

For very long acoustic waves, $qL < 1$, one has obviously to consider the motion of the sample per se, to account for the way the sample is held. The relevant equation of motion in the absence of the magnetic field is

$$J\omega^2 - C = 0$$

where $J$ is the moment of inertia and $C$ measures the rigidity of the sample holder. Adding to it the magnetic torque $\mathbf{K} = \mathbf{M} \times B_0$:

$$-\omega^2 J + C = \mathcal{K} B_0^2 V$$

(3.5)

where $V$ is the volume of the sample and $\mathbf{M}$ is the magnetisation.

Let the sample be fixed softly, for instance, using an elastic pivot. Then the first term in (3.5) is negligible at low frequencies and one obtains

$$\mathcal{K} B_0^2 V = C.$$ (3.6)

This case has been treated by Delaney and Pippard (1971, 1972). Equation (3.6), supplemented by (3.4), determines the vibrational frequency of the so-called ‘soft’ helicons. In the opposite case of the sample fixed rigidly in the sample holder, which in turn is fixed inside the cryostat with rigidity $C^*$, the sample and sample holder make a rigid body. Hence one has to replace $J$ and $C$ in (3.5) by the effective moment of inertia $J^*$ and by $C^*$. As $J^*$ can be large, the inertial term in (3.5) is not negligible.

The peak positions, calculated using (3.3) and (3.5), are compared with the experimentally observed positions in figure 10(c). Also shown are shifts from the helicon resonance field $B_1$, calculated according to (3.1):

$$\Delta B_r = B_r - B_1.$$ (3.7)

In figure 10(c) those curves with $\Delta B_r > 0$ correspond to the flexural waves, whereas those with $\Delta B_r < 0$ correspond to vibrations of the sample as a whole. Calculations of the satellite resonance positions using (3.3) and (3.5) show that, for higher-order resonances, they are too close to the main helicon resonances to be resolved under the experimental conditions prevailing.

The possibility of acoustic phenomena accompanying the helicon resonance should be kept in mind, in particular when the helicon resonance is used as a tool to study the properties of metals. Acousto-helicon effects may result, if satellites are not resolved, in the apparent non-linear dependence of the mean frequency of the observed peak on the magnetic field. One may be tempted to interpret this as the Hall constant being a function of the magnetic field. The related effect is the non-linear dependence on the magnetic field of the frequency of the helicon generator.

The acoustic effects are weaker in the vicinity of higher-order resonances (figure 9(c)). Lausmaa et al (1983) have made use of this by supplementing the helicon
generator with a variable filter (high-pass filter), thereby suppressing unwanted low-order resonances and making the system oscillate at any one of the higher-order resonances (shown by the broken contour in figure 7(i)). This enables one to reduce the non-linear effects by a few orders of magnitude and permit improved measurements to be made using a helicon generator.

In order to eliminate the acousto-helicon effects completely, one has to fix the sample as rigidly as possible and to make the bending-induced tangential component of the magnetic field (3.2) much smaller than the excitation field: \( h_{\text{max}} = B_0 \theta_{\text{max}} \ll h_0 \). For precision measurements one has to monitor the amplitude of acoustic vibrations directly.

We have discussed the influence of macroscopic acoustic phenomena of a surface nature on helicon resonance in metals. There also exist extensive studies of the interaction of helicons with the bulk sound, i.e. of the helicon–phonon interaction. This is an expanding field and one can find references on the subject in papers by Viswanathan (1975), Jdiculla and Viswanathan (1979, 1981) and Gudkov (1982).

3.3. Dopplerons

Dopplerons (see § 2.1.3) can be observed experimentally using the same techniques as used to observe helicons (§ 3.1). In the symmetric geometry (§ 2.3.1) one excites in the metal the standing doppleron wave resonances. The resonance frequencies are given by equation (2.86), where \( k \) is the wavenumber of the doppleron. As discussed above, the doppleron dispersion relation depends strongly on the FS topology and for a given metal one follows the derivation as for (2.52).

Since in the domain of existence of dopplerons one has an approximate relationship:

\[ kv_{\theta_{\text{max}}} \approx \omega_c \]

at the resonance in view of (2.86) one obtains

\[ (2n-1) \frac{\pi}{d} v_{\theta_{\text{max}}} \approx \frac{eB_n}{m_c c}. \]

Thus, one finds a sequence of resonance singularities almost periodic in the field. Such singularities have been observed experimentally by many authors (e.g. Weisbush and Libchaber 1967, Naberezhnykh and Maryakhin 1967, Krylov 1968a, b, Wood and Gavenda 1970, Perrin et al. 1970, Antoniewicz et al. 1968), although it took some time to understand their origin.

The reason is that in many respects dopplerons resemble in their experimental manifestations another interesting phenomenon, the so-called Gantmakher–Kaner oscillations (GKO), first discovered in 1965, well before dopplerons were observed. Now it is well understood that both phenomena do exist simultaneously, but for a long time doppleron resonances were confused with GKO.

Briefly, GKO correspond to singularities in the surface impedance of the metallic plate oriented perpendicular to the magnetic field, when the anomalous skin-effect criterion holds, namely the mean free path of electrons exceeds both the skin depth and the plate thickness, \( d \). Singularities take place if an extremely large group of electrons has the same displacement \( u_{\text{ext}} \) along the magnetic field per cyclotron cycle and if the plate thickness equals an integer number of displacements:

\[ nu_{\text{ext}} = n(2\pi v_z/\omega_c)_{\text{ext}} = d. \]
Singularities of the surface impedance are periodic in the magnetic field with period:

$$\Delta B_{\text{GK}} = \frac{2\pi c}{ed} |v_z m_{\text{ext}}|.$$  \hfill (3.9)

Sometimes this phenomenon is called the normal field radio-frequency size effect.

The principal experimental manifestations of GKO are as follows.

(i) Exact periodicity as a function of the magnetic field with the period given by (3.9).

(ii) A weak dependence of oscillation amplitude on the magnetic field under conditions of an anomalous skin effect.

(iii) Absence of a weak-field threshold.

(iv) Presence of the effect at any polarization of the electromagnetic wave in the skin layer.

(v) Relatively small amplitude, as the effect is proportional to the fraction of time an electron spends in the skin layer, \( t \sim \delta/l \), where \( \delta \) is the skin depth and \( l \) is the mean free path.

Oscillations of the surface impedance, possessing all the above features, were observed in potassium by Libchaber et al (1970). Similar oscillations have also been observed in indium, copper, aluminium, cadmium and some other metals. However, in most cases oscillations did not possess all the features of GKO.

Weisbush and Libchaber (1967) in copper and Krylov (1968a,b) in indium have discovered that the oscillation period depends on the magnetic field. Furthermore, they have observed the threshold behaviour of oscillations in magnetic field.

Fisher et al (1971) (see also Naberezhnykh and Tzimbal (1971), Naberezhnykh et al (1972) and Lavrova et al (1974)) have produced a comprehensive proof that, in most metals, the observed oscillations of the surface impedance in the region of DSCR of helicons (2.33) are, in fact, resonances due to the excitation of the standing doppleron.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure11.png}
\caption{Doppler-shifted cyclotron resonance and dopplerons \((d = 0.25 \text{ mm}, f = 167 \text{ kHz}, T = 4.2 \text{ K})\). \(a\) Helicon resonances and DSCR in copper (circular polarization of excitation field coincides with the direction of the Larmor precession of the charge carriers). Curve 1, the measured signal is proportional to the magnetic-field derivative of the surface impedance of the parallel-plate sample; curve 2, the same as above for the opposite polarization, standing-wave doppleron resonances are excited; \(b\) splitting \(\Delta B\) of doppleron resonances. \(\Delta B_{\text{GK}}\) is the period of the Gantmakher–Kaner oscillations. The theoretical \((\Delta B/\Delta B_{\text{GK}})\) dependence on \(B_0\) is shown by the thick curve. Figures are from Lavroba et al (1974).}
\end{figure}
Helicon resonance in metals

waves. The peculiar features of these waves, which enable one to distinguish them from GKO (see above), are as follows.

(i) The splitting of resonances, $\Delta B$, depends both on the magnetic field and excitation frequency.

(ii) Dopplerons exist in a limited region of magnetic fields in the vicinity of DSCR and show a sharp threshold onset as the magnetic field increases.

(iii) The observed amplitude of the oscillations is comparable to the helicon resonance amplitude and is, by a few orders of magnitude, larger than that of the size effects.

(iv) Dopplerons exist for only one of the two possible circular polarisations.

In figure 11(a) we present a typical experimental record of doppleron resonances in copper by Lavrova et al (1974). Note that in this case the circular polarisation of the doppleron is opposite to that of the helicon, which simplifies their separation. The signal measured is proportional to the magnetic-field derivative of the real part of the surface impedance. In figure 11(b) (Lavrova et al 1974) the experimentally observed and theoretically calculated magnetic-field dependence of the doppleron resonance period are compared. The theoretical curve was calculated for the known FS of copper and is in excellent agreement with experiment.

3.4. Resonance mechanisms of helicon damping

A characteristic feature of resonance damping mechanisms (Landau damping, Doppler-shifted cyclotron resonance) is their strong dependence on the wavelength, whereas the 'background' collision damping is wavelength-independent. This makes it possible to separate them experimentally. DSCR has a sharp edge below which the propagation of helicons is impossible (figures 8(a) and 11(a)). The relevance of the edge to DSCR can be verified via the relationship for the edge field $B_E$ and excitation frequency:

$$\omega \sim B_E^3.$$  \hspace{1cm} (3.10)

The third-power law can be derived easily from the resonance condition (2.33) and has been checked in numerous experiments (see, for example, Taylor et al (1963a,b), Libchaber and Grimes (1969) and Lavrova et al (1974)).

Far from the DSCR edge, LD and collision damping dominate, which can be separated by studying the wavelength dependence of the damping. One can define the collision damping as a 'residual' damping in the limit of $kl \rightarrow 0$, determined experimentally, for example, by extrapolation of measurements of the $Q$ factor of helicon resonances in a parallel-plate sample as a function of the resonance order. Since for such a measurement $k l \approx \pi l / d$, an accurate determination of the collision damping requires the use of a sample with thickness large compared to the mean free path. If the collision damping dominates, then the $Q$ factor should show practically no dependence on the resonance order$^\dagger$.

Shown in figure 12(a) are typical experimentally observed angular dependencies of total helicon damping at different wavelengths and electron mean free paths (a larger mean free path corresponds to a lower temperature) for a metal with an anisotropic FS (indium). The following features are of interest.

$^\dagger$ We stress that in samples of other shapes, e.g. spheres, the $Q$ factor depends on the resonance order even if collision damping dominates, since the geometry of current flow depends essentially on the resonance order.
At higher temperatures the wavelength dependence of helicon damping is much weaker than at lower temperatures. The reason is that at higher temperatures the electron mean free path is small due to the scattering on phonons, so that one has the local limit $k\lambda \ll 1$ both for the fundamental resonance ($n = 1, \lambda = 1.4$ mm) and the second harmonics ($n = 2, \lambda = 0.47$ mm). Collision damping dominates.

As the wavelength is lowered, there is an extra Landau damping with a specific non-monotonic dependence on the angle between the magnetic field and the helicon propagation direction. Positions of the resulting minima of $LD$ are in reasonable agreement with those calculated using the procedure of figure 5. According to the latter, the damping is minimal for the magnetic field, directed perpendicular to the lines $\gamma_1\gamma_2'$ and $\gamma_3\gamma_4'$ (see figure 12(c)), in agreement with experiment. The above procedure does not work for the third minimum. A similar angular dependence of the $LD$ was observed by Krylov (1968a, b) and Halevi et al (1969).

For certain orientations of the magnetic field $LD$ vanishes, for example at $\theta = -25^\circ$ in figure 12(a). In this case variations of the damping are due to the temperature dependence of the transport relaxation time $\tau$ (see (2.74) and (2.75)) and decrease monotonically as the temperature decreases. At other orientations the temperature dependence is determined by competing collision and Landau damping (see curves in figure 12(b)). The contribution of the latter increases due to an increase of the mean free path of electrons at low temperatures. Under these conditions both the total damping (figure 12) and $LD$ (Volskii and Petrashov 1973) can possess a non-monotonic dependence on temperature.

A non-monotonic temperature dependence of the damping of dopplerons having the same origin as that of helicons was also observed experimentally (Voloshin et al 1978).

The results of the measurements of $LD$ and $DSCR$ in metals with spherical FS (e.g. measurements in K by Houck and Bowers (1968) and Libchaber and Grimes (1969)) correspond perfectly to the theory presented in §§ 2.1.3 and 2.2.1.
3.5. Helicons in the local limit: the electronic properties of metals

The first practical application of helicons was the electrodeless measurement of the Hall constant. It can easily be computed from equations (2.91) and (2.92), once the resonance magnetic field and frequency have been measured in a parallel-plate sample of known thickness and orientation to the magnetic field (Chambers and Jones 1962) (see also references at the end of § 3.1). Important information on the metal can be deduced from the measurements of the damping of helicons in the local limit.

3.5.1. Investigation of open Fermi surfaces. Grimes et al (1965) were the first to study experimentally the influence of open orbits on the propagation of helicons. They observed a steep rise of the damping at magnetic-field orientations for which open orbits exist. At the maximum of the damping the spectrum of helicons deforms and transforms into damped Alfvén waves.

The first observations by Grimes et al (1965), and further work by Merrill (1968), were at the qualitative level. More detailed studies were carried out by Hui (1969) in copper and by Giovanielli and Merrill (1970) in silver and yielded quantitative information on open orbits in these metals.

The sensitivity of the helicon resonance amplitude, $Q$ factor and frequency to the presence of open orbits can be seen in figure 13(a) (from Giovanielli and Merrill 1970) on helicon resonances in a parallel-plate sample of silver. The fractional variation of the damping of helicons is considerably greater than that of the frequency.

![Figure 13](image-url)  
**Figure 13.** Helicons in a metal with an open FS (silver) (from Giovanielli and Merrill 1970). (a) Resonance frequency $f_r$, $Q$ factor and the resonance amplitude $A$ as a function of the angle between the magnetic field $B_0$ (30 kG) and the opening-free direction $[110]$. Variations of $f_r$ are insignificant compared to those of $Q$ and $A$; (b)–(d) the fraction of electrons on open trajectories at different orientations of the magnetic field, measured employing helicons.

From an analysis of the experimental data, Giovanielli and Merrill (1970) have determined the relative fraction of electrons in silver, on open orbits, as a function of the angle between the magnetic field and crystal axes (see figures 13(b), (c) and (d)). In this work the majority of the components of the magnetoresistance tensor have been studied in detail.

3.5.2. Study of conduction electron scattering. As mentioned in § 2.2.2, in metals with a closed FS the collision damping measurements enable one to determine the transport
relaxation time in a strong magnetic field. Apart from the above-mentioned advantages of the helicon technique—electrodeless measurements, high sensitivity due to the resonance character of signals measured, etc.—of no less importance is that the mere presence of helicon resonance is evidence of the strong-field condition $\omega_c \tau \gg 1$. One can thus use the simple asymptotic formulae (2.71).

Whatever the scattering mechanism, for a random distribution of static scattering centres, the transport scattering rate of electrons $\tau^{-1}$ and, consequently, the damping of helicons are proportional to the concentration of scattering centres. Making use of formulae like (2.28), which relate the mean free path to the scattering centre cross section and concentration, we obtain

$$\Gamma_c = (\Phi(\theta, \varphi) / v_F / B_0 \cos \theta) N_{\text{imp}}. \quad (3.11)$$

An experimental check on the linearity of $\Gamma_c$ as a function of $N_{\text{imp}}$, when impurity scattering dominates, was carried out by Volskii and Petrashov (1974) and Bronnikov et al. (1974). Single-crystal indium samples having concentrations of lead of $10^{-4}$-5×$10^{-3}$ at% were used. Particular attention was paid to determining the concentration and distribution of impurities in the bulk of the samples using electrochemical and x-ray methods. The experimentally measured damping of helicons in indium is plotted against the lead concentration in figure 14(a). The damping of helicons is enhanced approximately thirty-fold after adding $5 \times 10^{-3}$ at% of lead to the initial sample.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure14}
\caption{Collision damping of helicons. (a) $\rho_M = \Gamma B_0 R_M$ as a function of the lead content in indium ($R_M = 1.56 \times 10^{-12} \ \Omega \ cm \ G^{-1}$), (b) anisotropy of collision damping in indium for different types of scatterers.}
\end{figure}

As the Fermi velocity and characteristic cyclotron mass of the dominating group of electrons in indium are known ($m_e = 2m_0$, $v_F = 1.1 \times 10^8$ cm s$^{-1}$, Mina and Khaikin (1965, 1966)), the transport cross section of the scattering of conduction electrons on lead atoms, dissolved in indium, may be determined from (3.11) as $\langle \sigma \rangle = 2 \times 10^{-16} cm^2$. This agrees well with the screening of impurity potentials at distances of the order of

\footnotetext[1]{Analogous determination is possible in the presence of open orbits as well, if one uses formulae similar to (2.83) with known $rs$ parameters.}
\footnotetext[2]{See § 3.6 for a discussion of possible conductivity anomalies in a strong magnetic field.}
the interatomic distances in metals (see, for example, Kittel (1963)), and demonstrates that the helicon resonance is a potential tool for studying electron–impurity interactions.

Important information on the relaxation mechanisms and on transport relaxation times can be deduced from measurements of the anisotropy of the damping of helicons. Such studies (Petrashov and Heim 1981) have led to an experimental proof of the sizable anisotropy of the transport relaxation time $\tau$ of electrons in a strong magnetic field.

Recently there have been numerous studies of the anisotropy of the relaxation time of conduction electrons and some comments are in order here. Particularly impressive are the successes in studies of the relaxation time $\tau(p)$ of groups of electrons, occupying small domains on the Fermi surface. The anisotropy of $\tau(p)$, i.e. the dependence of the relaxation time of the small group of electrons on its position $p$ on the Fermi surface, has been studied (see, for example, reviews by Gantmakher (1974) and Springford (1980)). However, one has to bear in mind that the time $\tau$, which characterises the damping of helicons, is characteristic of the relaxation time of the electronic system of the metal as a whole and does not necessarily have a simple description in terms of the relaxation times of individual electrons and/or small selected groups of electrons.

Such a situation occurs when phonon scattering predominates at low temperatures (see the review by Gurzhi and Kopeliovich (1981)). The momentum of the electron system of the metal, fed by the electric field, is transferred as a result of collisions to the phonons. It may happen that phonons drift in the electric field alongside the electrons. If there is no dissipation of the momentum from the electron–phonon system, such a drift of electrons and phonons persists even after the electric field is switched off and implies a no-relaxation situation, $\tau \to \infty$. If a group of electrons out of equilibrium is sufficiently small its total momentum, gained from the electric field, is not large enough to carry along all the electrons and phonons. Hence such a group would always have a finite relaxation time. This illustrates nicely that measurements of the transport relaxation time and of its anisotropy are of interest in themselves.

Shown in figure 14(b) are the experimental results for the anisotropy of collision damping in indium for the cases of dominance of the scattering by impurities (two lower curves) and by phonons (the upper curve) (Petrashov and Heim 1983). Measurements were carried out with single-crystal spherical samples of 10 mm in diameter, which were grown in a demountable quartz mould.

Although indium has an anisotropic closed FS, the spectrum and polarisation of helicons in a strong magnetic field are practically the same as in metals having a spherical FS (see § 2.1.2), so that the position and shape of the resonances are well described by the Werner and Ford (1975) formulae (§ 2.3.2). For isotropic scattering of electrons, i.e. if $\tau$ in (2.71) does not depend on $p_z$, one can write an approximate relation following (2.97):

$$Q^{-1} = \frac{\langle (\Theta(\theta, \phi)/\cos \theta) / B_0 \rangle}{\sqrt{2}} \tau_{\text{eff}}^{-1} = 4 \tau_{\text{eff}}^{-1} / \omega_c.$$  (3.12)

Here the bar implies an averaging over $\theta$, the angle between the magnetic field and the normal to the sample surface. The $Q$ factor of helicon resonances in a sphere depends only on the angles between the magnetic field and crystal axes.

As seen from figure 14(b), when impurity scattering dominates, the anisotropy of collisional damping is independent of impurity concentrations. (The curve for the highest purity sample is slightly asymmetric because the sample was slightly plastically deformed.) This can easily be understood, as the ratio $\Gamma_c / N_{\text{imp}}$, according to (3.11),
does not depend on concentration and, for a fixed magnetic field, is a universal function of orientation$. The form of the anisotropy may well depend on impurity type, but experimental studies of this are lacking.

When anisotropic electron–phonon scattering dominates the result is qualitatively different (figure 14(b)). To eliminate spurious effects caused by the trivial change of the field-strength criterion from one kind of scatterer to another, one has to compare anisotropies at the same value of parameter $\omega_c \tau_{\text{eff}}$ (see figure 14(b)).

Were the anisotropy of helicon damping due, not to the anisotropy of the relaxation time $\tau_{\text{eff}}$, but rather to the FS topology, then all the curves in figure 14(b) would be similar to each other, irrespective of the temperature and impurity concentration and would depend only on the function $\Phi(\theta, \varphi)/\cos \theta$ in (3.12). In fact, the curves can only be understood in terms of an anisotropy of the transport relaxation time $\tau_{\text{eff}}$ of conduction electrons in formulae (2.75) and (3.12).

The anisotropy due to the electron–phonon scattering can qualitatively be explained within the framework of the Gurzhi–Kopeliovich (1974, 1981) theory. This is based on the detailed consideration of the quasi-momentum conservation in the electron–phonon system and of phonon drag effects.

At low temperatures, $T \ll T_D$ ($T_D$ is the Debye temperature), the momentum transfer in electron–phonon scattering is almost perpendicular to the electron’s velocity (Gantmakher 1974), so that the electron remains on the Fermi surface. Collisions are uncorrelated, with typical displacement $q = KT/\hbar s < p_F$ ($s$ is the sound velocity and $K$ is the Boltzmann constant). Hence one encounters diffusion on the Fermi surface (Klemens and Jackson 1964). For a closed FS, of particular importance are the so-called hot spots, where two FS are close to each other in momentum space (Pippard 1968, Young 1968) (figure 15(a)). If phonon momentum $q$ exceeds the separation $\Delta p$ between two adjacent FS, hops from one FS to another may become possible. Such a hop can take place with the momentum transferred to the lattice as a whole (a U process) (e.g. Gantmakher 1974). The U processes cause a dissipation of the momentum of the electron–phonon system at low enough temperatures, lead to a finite transport relaxation time and hinder the phonon drag.

The case of crossed electric and magnetic fields with a Hall-type non-equilibrium component in the electron distribution function is shown in figure 15(a). Gurzhi and Kopeliovich (1974) have observed that, in this case, U processes in the geometry of figure 15(a) change the total momentum of the electron system in the direction of the magnetic field. This change should be compensated by an electron drift along the magnetic field since, in such a geometry, the electric field does not affect the momentum along the magnetic field. Besides this, Gurzhi and Kopeliovich have taken into account a compensation of the diffusive flow of electrons and U-process-induced flow in momentum space, implied by particle number conservation in momentum space. In view of this, diffusion and U processes are closely interconnected.

Gurzhi and Kopeliovich (1974) have solved the relevant kinetic equations and have found that the Hall components of conductivity $\sigma_{xy}$ and $\sigma_{yx}$ are not affected by the scattering of electrons and are still described by formulae (2.71) and (2.72), whereas the dissipative components, for a sufficiently small number of small hot spots, can be described by the effective relaxation time $\tau_{\text{eff}}$:

$$\sigma_{yy}, \sigma_{xx} = (ne^2/m\omega_c^2)\tau_{\text{eff}}^{-1}. \quad (3.13)$$

† The anisotropy may change in sufficiently strong fields if the magnetic breakdown produces open orbits (Petrashov and Springford 1983).
Here the effective relaxation time $\tau_{\text{eff}}$ is a period of motion of an electron in the cycle $A \rightarrow U$ process $\rightarrow A'$ diffusion $\rightarrow A$ (figure 15(a)). It equals the sum of the time $\tau^b_D$ of diffusion of an electron across a belt of width $b$ on the Fermi surface (figure 15(a)) and the inverse rate of transitions from one FS to another, $\tau^r_U$:

$$\tau_{\text{eff}} = \tau^b_D + \tau^r_U. \quad (3.14)$$

Here $\tau^b_D = \tau_F b/p_F$, where $\tau_F$ is the diffusion time for traversing a distance of the order of the Fermi momentum $p_F$, $\tau_F \sim T^{-5}$, $\tau^r_U = (p_F^2/r_0^2) \tau_U$, where $r_0$ is the hot spot radius and $\tau_U^{-1}$ is the U-process transition rate. For $q < \Delta p$ the U-process rate should decrease exponentially as the temperature decreases (e.g. Gurzhi and Kopeliovich 1981).

If the electron–phonon scattering dominates the damping of helicons then

$$\Gamma = \tau_{\text{eff}}^{-1}/\omega_c = \left\{ \omega_c \left[ (b/p_F)\tau_F + (p_F^2/r_0^2)\tau_U \right] \right\}^{-1}. \quad (3.15)$$

There are two sources of damping anisotropy: anisotropy of the cyclotron frequency and of $\tau_{\text{eff}}$. An important observation is that $\tau_{\text{eff}}^{-1}$ is maximal at $b \rightarrow 0$, i.e. when hot spots between which U processes take place (equivalent hot spots) lie in the same plane perpendicular to the magnetic field.

The above is exemplified by the anisotropy of $\tau_{\text{eff}}^{-1}$ in the (011) plane of indium (figure 15(b)), measured using helicons (Petrashov and Heim 1983). A contribution from the anisotropy of $\omega_c$ is eliminated by taking the ratio $\Gamma_{\text{phonon}}/\Gamma_{\text{imp}}$, where $\Gamma_{\text{phonon}}$ and $\Gamma_{\text{imp}}$ are the electron–phonon and electron–impurity damping, respectively.

Points W and T may act as hot spots in indium at temperatures 1.3–4.2 K, at which the measurements were taken (figure 15(b)). The anisotropy of $\tau_{\text{eff}}^{-1}$ (figure 15(b)) can be understood qualitatively in terms of the geometry of hot spots, between which hops on the FS are possible (Petrashov and Heim 1983).

There exist rather accurate models of the FS of indium, so that hopefully one can treat quantitatively the experimental results in terms of characteristics of electron–phonon scattering and phonon drag.
The influence of temperature and impurities on the damping of helicons was also studied by Volskii and Petrashov (1973) and Bronnikov et al (1974) in indium and by Janssen and Witters (1975a, b) in aluminium. Comparisons of the helicon damping and zero-field conductivity on the same samples enabled one to conclude that the relaxation of the conduction electrons in strong and in zero magnetic fields differs.

The experimental observation that the damping of helicons is proportional to the impurity concentration at low enough temperatures has led Volskii and Petrashov (1974) to propose the use of helicon resonance to measure the purity of metals, as an alternative to the measurement of the residual resistivity ratio in zero magnetic field. Such measurements in a magnetic field are particularly advantageous when one has to estimate the purity of superconducting metals.

In § 5 we shall again discuss the influence of the scattering of electrons on the propagation of helicons in connection with quantum effects.

3.6. Anomalous propagation of helicons in potassium

The propagation of helicons in potassium possesses a few remarkable, and as yet unexplained, features. On the one hand, according to measurements using different methods, the FS of potassium is spherical with radius variations not exceeding 0.1% (Shoenberg and Stiles 1964, Lee and Falicov 1968, Blaney and Parsons 1970). The propagation of helicons in the non-local regime agrees well with such a picture (Libchaber and Grimes 1969). On the other hand, many of the properties of potassium conflict with predictions based on the premise that the FS is spherical (for a review see Overhauser (1978) and also Coulter and Datars (1980)).

It is well established that the collision damping of helicons in potassium contains the anomalous component $\Delta \Gamma$, weakly dependent on magnetic field:

$$\Gamma = \left( \omega_c \tau \cos \theta \right)^{-1} + \Delta \Gamma$$  \hspace{1cm} (3.16)

(compare with (2.55)). The presence of $\Delta \Gamma = \text{constant}$ amounts to the linear magnetic-field dependence of the transverse magnetoresistivity, $\rho_T \sim \rho_{xx} \rho_{yy}$, in a strong magnetic field (Penz and Bowers 1968), which grossly contradicts the present electron theory of metals (Lifshitz et al 1956) (LAK), carefully checked in numerous other experiments. We note that formulae (2.71) correspond to the LAK theory. Furthermore, the helicon resonance frequency in potassium possesses a non-linear dependence on magnetic field. This can be interpreted as the Hall constant being a function of the magnetic field (Penz and Bowers 1968), which again contradicts the LAK theory. Additionally, there are surprisingly strong anisotropies of the $Q$ factor and frequency of helicon resonance in spherical samples of potassium (O'Shea and Springford 1981).

A characteristic feature of all the above anomalies of the electron properties of potassium is their irreproducibility after heating the samples up to room temperature (e.g. Overhauser 1978). A possible explanation of these anomalies essentially supposes a non-spherical shape of the Fermi surface (e.g. Overhauser 1978, Huberman and Overhauser 1981). If so, all the experimental observations could be made consistent with LAK theory. However, simultaneous precision measurements of the FS and helicon parameters (O'Shea and Springford 1981) have confirmed the sphericity of the FS while anomalies persist in the same potassium samples. Therefore, more studies of the anomalous properties of potassium are necessary.

In addition we note that the linear magnetoresistance may also be observed in some other metals with a closed FS, for instance aluminium and indium. However, it
can, to a large extent, be eliminated by reducing the concentration of dislocations, macroscopic imperfections, etc (Delaney 1974, Amundsen and Jestard 1974).

3.7. Helicons in the intermediate state of type-I superconductors

The Hall effect persists in the mixed state of type-II superconductors and in the intermediate state of type-I superconductors. In these cases the origin of the Hall effect is different from that of the Hall drift in crossed fields in normal metals (de Gennes and Nozieres 1964, Andreev 1966), but still the macroscopic relationship between the field and current retains its conventional form, so that propagation of helicon-like waves is possible. Indeed, the propagation of circularly polarised waves has been observed experimentally both in type-II (Druyvesteyn et al 1966) and type-I superconductors (Maxfield and Johnson 1965).

For a long time it was not easy to compare the experimental results with the theory in type-I superconductors, since they have very low critical fields and there existed no sufficiently pure metals to satisfy the strong-field condition \( \omega_c \tau \gg 1 \). Recently such experiments were successfully performed (Krylov et al 1974, Krylov 1976).

Helicons with wavelengths exceeding the size of the normal-phase and superconducting domains have been studied. The latter situation has been considered theoretically by Andreev (1966). Since one averages over large scales, the resulting formulae have only a weak dependence on the specific structure of the intermediate state and on the shape and velocity of motion of domains. The measured parameters proved to depend only on the external magnetic field and to be independent of the sample magnetisation history, whereas the structure of the intermediate state itself is known to depend, for example, on the magnetic-field sweep direction (Haenssler and Rinderer 1965, Livingston and De Sorbo 1969).

The geometry of fields inside the plate of metal in the intermediate state has certain peculiarities. Inside the plate in the intermediate state the modulus of the magnetic field equals the critical field: \( |H_0| = H_c \), whereas the angle \( \theta \) between the internal field \( H_0 \) and normal to the plate depends both on the external field \( H \) and its inclination \( \varphi \) to the normal to the plate:

\[
\sin \theta = \left( \frac{H}{H_c} \right) \sin \varphi.
\] (3.17)

Helicon resonances in such a plate are described by formulae analogous to (2.91) and (2.92). However, the detected signal becomes a function of the concentration of the normal phase \( C_N \):

\[
U_2 = i \omega \mu G C_N J_1
\] (3.18)

where

\[
C_N = \left( \frac{H}{H_c} \right) \cos \varphi \left[ 1 - \left( \frac{H}{H_c} \right)^2 \sin^2 \varphi \right]^{-1/2}.
\]

The angular dependence of the helicon resonance parameters differs from that for the normal metal (Krylov 1976). At \( \sigma_{xx} \ll |\sigma_{xy}| \) one finds

\[
Q^{-1} = \sigma_{xx} / |\sigma_{xy}| = 2 R_H H_c / \rho_T
\]

\[
\omega_1 = (c^2 R_H H_c \cos^2 \theta / 4 d^2)(1 + 2.1 / Q^2)
\] (3.19)

where \( R_H \) and \( \rho_T \) are the Hall constant and transverse magnetoresistivity of the normal metal.
In figure 16(a) we show some of Krylov's results (Krylov 1976) on an experimental check of the angular dependencies (3.19). An excellent agreement with theory was found. The $Q$ factor of the resonance in a plate depends weakly on the angle and the resonance frequency is proportional to $\cos^2 \theta$ rather than to $\cos \theta$.

The damping of helicons decreases after the transition into the intermediate state, which may be interpreted as the corresponding increase of effective mean free path of electrons (figure 16(b)) (Krylov et al 1974). They attributed the effect to the scattering of electrons on the sample boundary. Indeed, electrons localise inside the normal-phase domains and interact less frequently with the sample-vacuum boundary, whereas the so-called Andreev scattering on the normal-phase-superconductor boundary does not contribute to the dissipative conductivity.

4. Theory of helicons in the presence of Landau quantisation

At low temperatures and in a sufficiently strong magnetic field practically all the kinetic and thermodynamic characteristics of metals are oscillating functions of the magnetic field (see, for example, Cracknell and Wong (1973)). These oscillations are of quantum origin and are due to quantisation of the conduction electron energy in a strong magnetic field:

$$e(n, p_z) = \hbar \omega_c (n + \frac{1}{2}) + \frac{p_z^2}{2m}$$

(4.1)

where $n$ is an integer.

Quantum levels with different $n$ in (4.1) are called the Landau levels. The quantum oscillation phenomena are sometimes called simply the Landau oscillations.

A well-known example of such oscillations are those of the static diagonal components of the conductivity tensor (2.71): the so-called Shubnikov–de Haas effect. According to formula (2.54), the Shubnikov–de Haas effect should result in oscillations in the damping of helicons.

Grimes (1964), who was the first to observe quantum oscillations in helicon propagation, considered this to be the explanation of his results. However, the magnitude of the effects that he observed was much larger than that expected, based on the theory of the Shubnikov–de Haas effect. The problem is that in static conductivity
the quantum oscillatory terms are proportional to a small factor \((\hbar \omega_c/e_F)^{1/2} \approx n^{-1/2}\) (Adams and Holstein 1959) (see also the review by Landwehr (1969)). Since in real cases \(n \approx 10^2\)–\(10^4\), the amplitude of quantum oscillations in the helicon propagation should not exceed a few per cent. The expected effect is lowered further if one takes into account peculiarities of the FS of aluminium, in which the oscillations were observed by Grimes (Parker and Balcombe 1968).

In subsequent theories (Miller and Kwok 1967, Kaner and Skobov 1968) Grimes' results were still interpreted as being due to oscillations of dissipative components of the conductivity tensor (2.71), but rather non-local ones, connected with the magnetic Landau damping. Their origin was presumed to be in oscillations of the number of electrons having the resonance velocity \(v_0\) in the quantum counterpart of (2.59), around the mean 'classical' number. In fields for which the resonance velocity is excluded by the quantisation conditions, the Landau damping is reduced and it is enhanced if the resonance velocity is accessible. Such oscillations, in principle, may be large, the 'giant' ones (Kaner and Skobov 1968).

Neither of the above mechanisms is relevant to experimentally observed quantum oscillations in the helicon propagation (Volskii and Petrashov 1968). Volskii and Petrashov have proved their origin to be a novel one: an oscillatory phase velocity of helicons rather than their damping. Therefore, the oscillating component should rather be included in the dispersion relation (2.38). Similar, but less conclusive, evidence was almost simultaneously obtained by Krylov (1968b).

Volskii and Petrashov (1968, 1970) have derived the dispersion equation with allowance for quantum oscillations. Oscillations in helicon propagation were related to the de Haas–van Alphen effect, quantum oscillations of the differential magnetic susceptibility. Okulov (1970) has started with the microscopic kinetic equation with allowance for quantum magnetisation currents. His results for the oscillatory terms in the dispersion relation coincide with those of Volskii and Petrashov (1968, 1970). Similar conclusions were reached by McIntyre and Quinn (1970).

de Haas–van Alphen effects in the dispersion relation and in the propagation of helicons have been checked experimentally by Volskii and Petrashov (1968, 1970), Amundsen and Jestard (1972), Balibar et al (1972), Bozhko and Volskii (1975) and others and there is good agreement between theory and experiment.

A feature of the de Haas–van Alphen effect is that, in a perfect pure metal crystal, the amplitude of the oscillations in the differential magnetic susceptibility increases without limit as the temperature is lowered and the magnetic field is decreased. At certain amplitudes one can observe qualitatively new phenomena, not accessible in the 'classical' limit. We cite here the onset of thermodynamic instabilities and diamagnetic domains in metals (Shoenberg 1962, Pippard 1963, Condon 1966). Simultaneously one observes large oscillatory effects in the propagation of helicons (Zherebchevskii and Kaner 1969, Volskii and Petrashov 1971).

Note that phenomena connected with the de Haas–van Alphen effect should persist both in the local, \(kl \ll 1\), and non-local, \(kl > 1\), limits. In the non-local limit one encounters a complex superposition of effects due to oscillations of the magnetic susceptibility and, in principle, of the non-local conductivity as well, although no convincing experimental evidence for the latter exists to date. Apparently, this failure is because the conditions for their observation are much too restrictive (see below). Furthermore, the hybrid oscillations of non-local damping, connected with oscillations of the helicon wavelength due to the de Haas–van Alphen effect, are possible as well (Volskii and Petrashov 1971).
4.1. Dispersion relation

The principal reason why the de Haas–van Alphen effect affects the propagation of helicons is that it makes the magnetic induction $b$ and the magnetic field $h$ of the wave distinct. The magnetic field $h$ of the wave is related to the magnetic induction $b$ and the induction $B_0$ of the de Haas–van Alphen effect by the relation:

$$h = b - 4\pi[M(B_0 + b) - M(B_0)].$$  (4.2)

In writing (4.2) we have taken into account that the magnetic moment $M$ is determined by the average microscopic field in the metal, i.e. by the induction $B$ (‘magnetic interaction’ or the Shoenberg effect (Shoenberg 1962)).

Following Lifshitz and Kosevich (1955), one can expand the oscillating magnetic moment as follows:

$$M = -m \sum_{r=1} A_r \sin[2\pi r(F/B) + \varphi].$$  (4.3)

Here $A_r$ is a smoothly varying function of the magnetic field $B$, of the temperature and the electronic parameters of the metal. $F = \hbar e S_{ext}/2\pi c$ is the oscillation frequency, $S_{ext}$ is an area of extremal cross section of the Fermi surface, normal to the field, $\varphi$, is a phase constant and $m$ is the unit vector of magnetisation $M$ in the crystal:

$$m_i = \alpha_i + [(\alpha_k^2 + \alpha_i^2)/F] \frac{\partial F}{\partial \alpha_i} - \frac{\alpha_i \alpha_k}{F} \frac{\partial F}{\partial \alpha_k}.$$  (4.4)

where $\alpha_i$ are components of the unit vector along $B$. If the magnetic field of the wave is smaller than a period of the de Haas–van Alphen oscillations, then the $h-b$ relationship (4.2) for the wave can be written as a linear algebraic one:

$$\frac{\partial M_i}{\partial B_k} \bigg|_{b=0} = \mu_{ik} b_k$$  (4.5)

where $\mu_{ik} = \delta_{ik} - 4\pi \frac{\partial M_i}{\partial B_k}$ and $\delta_{ik}$ is the Kronecker delta. By virtue of (4.3) derivatives $\chi_{ik} = \frac{\partial M_i}{\partial B_k}$, i.e. a static differential magnetic susceptibility, are oscillatory functions of the field $B$.

Inserting (4.5) into the Maxwell equation (2.3) and eliminating the magnetic induction of the wave and current using (2.2) and (2.8), we obtain for the wave equation:

$$\hat{\sigma} e = \frac{ic^2}{4\pi\omega} [(k \times (k \times e)] - 4\pi[k \times [\hat{\chi}(k \times e)]].$$  (4.6)

Comparing this with equation (2.12) in the classical limit, we see that the de Haas–van Alphen effect amounts to including the magnetisation current $j_m$:

$$j_m = \frac{ic^2}{\omega} \{k \times [\hat{\chi}(k \times e)]\}$$  (4.7)

which is of purely quantum origin.

One can easily derive the desired dispersion equation from the system of homogeneous equations (4.6) in the most general case. Still, it is useful to look for novel features, starting with the case of a spherical FS. In this case $M$ and $B$ are parallel to each other. The resulting system of equations is similar to (2.13), apart
from a modification of (2.13(a)) (as above, the coordinate system connected with the magnetic field is used (figure 3(a))):

\[
\begin{bmatrix}
\sigma_{xx} + (ic^2k^2/4\pi\omega) \left( 1 - 4\pi \frac{\partial M}{\partial B} \sin^2 \theta \right) \\
1 + \sigma_{xy} + \sigma_{xz}e_z = 0.
\end{bmatrix}
\] (4.8)

If \([1 - 4\pi(\partial M/\partial B) \sin^2 \theta] \neq 0\), the net effect is a renormalisation:

\[
\sigma_{ik} \rightarrow \tilde{\sigma}_{ik} = \sigma_{ik} \left( 1 - 4\pi \frac{\partial M}{\partial B} \sin^2 \theta \right)^{-1} i = x; k = x, y, z.
\] (4.9)

In terms of the renormalised components of the 'conductivity' expressed by (4.9), all the former general expressions (2.22) and (2.53) hold for the dispersion and damping of helicons. In the long-wavelength limit the dispersion equation takes the form:

\[
(\text{Re } k)^2 = \frac{4\pi\omega|\sigma_{yx}|}{c^2 \cos \theta [1 - 4\pi(\partial M/\partial B) \sin^2 \theta]^{1/2}}.
\] (4.10)

The damping (2.54) also acquires the oscillating term:

\[
\Gamma = \left[2 - 4\pi(\partial M/\partial B) \sin^2 \theta \right]/2[1 - 4\pi(\partial M/\partial B) \sin^2 \theta]^{1/2} (1/\omega_c \tau \cos \theta).
\] (4.11)

A polarisation too acquires an additional ellipticity, oscillating as a function of the magnetic field:

\[
es_y/e_x = i[1 - 4\pi(\partial M/\partial B) \sin^2 \theta]^{1/2} / \cos \theta \\
es_z/e_x \sim (\omega_c \tau)^{-1} \ll 1.
\] (4.12)

Hence, in the quantum case, practically all the characteristics of helicons become oscillating functions of the magnetic field. The oscillations of the damping are relatively smaller than those of the phase velocity. It is remarkable that the angular dependence of effects, including the lack of influence of quantisation at \(\theta = 0\), resembles that of the Landau damping (2.67). This is one of the reasons why the observed oscillations were erroneously taken for the Landau damping oscillations in early studies of the effect.

A detailed analysis of the dispersion equation in a general case of non-parallel \(M\) and \(B\) can be found in the papers by Volskii and Petrashov (1970), Okulov (1970) and Volskii (1975).

The oscillatory dependence of \(k\) on \(B\) and the oscillations in the polarisation should be taken into account in all the phenomena, discussed in §§ 2 and 3, such as DSCR, LD, acoustic effects, etc.

### 4.2. Resonances in parallel-plate samples

#### 4.2.1. Qualitative considerations.

An exact solution of the boundary value problem for excitation of helicon resonance under conditions of quantisation in an infinite plate was found by Volskii (1975). Before deriving the exact formulae, a qualitative discussion is in order. For symmetric excitation the resonance criterion (2.86) retains its form; resonances are excited once the plate thickness equals an odd number of half-wavelengths:

\[
\text{Re } k = (4\pi n_e \omega / cB_0 \cos \theta)^{1/2}[1 - 4\pi(\partial M/\partial B) \sin^2 \theta]^{-1/4} = (2n - 1)\pi / d.
\] (4.13)

The real part of the wavevector (4.10) is plotted against the magnetic field at fixed
frequency in figure 17(a). The resonance values of the signal in the pick-up coil correspond to intercepts of this curve with the straight lines $k = (2n-1)\pi/d$. The single intercept of the curve $k(B)|_{\omega=\text{constant}}$ with the straight line, corresponding to a fixed $n$ in the absence of oscillations (figure 7(f)), is replaced by a whole series of resonance signals. Moreover, on the $D-D'$ part of the curve there are two intercepts with the straight line $k = k_{res}$ per one oscillation period, so that a second harmonic with doubled oscillation frequency is generated. Variations of the signal amplitude can be determined graphically, plotting resonance curves a-a' and b-b' (figure 17(a), lower part), which correspond to intercepts of straight lines $k = k_{res}$ with envelope curves of oscillations of $k(B)|_{\omega=\text{constant}}$. One can take into account the oscillatory damping by recalling that the amplitude of the resonance signal can oscillate as well...
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(the part D–D' on the \( U_2-B_0 \) plot is not a straight line, but rather is an oscillating curve). More detailed considerations of the resulting patterns can be found in the paper by Volskii and Petrashov (1971).

As a consequence of quantisation the resonance frequency as well is also an oscillatory function of the magnetic field, unlike (2.91) and (2.92):

\[
\omega_n = (2n - 1)^2 (\pi/d)^2 (cB_0 \cos \theta/4\pi n e)[1 - 4\pi (\partial M/\partial B) \sin^2 \theta]^{1/2}. \tag{4.14}
\]

In the helicon generator experiments this should result in the oscillatory dependence of the generator frequency on magnetic field (Volskii and Petrashov 1968, 1970).

4.2.2. Solution of the boundary value problem—anisotropic Fermi surfaces. Following Volskii (1975), we choose the coordinate system \( OX'Y'Z' \) connected with the wavevector \( \mathbf{k} \) (figure 3(b)). Let the \( OX' \) and \( OY' \) axes be the principal axes of the magnetoresistivity tensor. Then the characteristics of the wave and the measured signals depend only on components of the two-dimensional part of a three-dimensional tensor \( \hat{\rho}' \) (Bass et al 1963):

\[
\hat{\rho}' = \begin{pmatrix}
\rho'_{xx} & \rho'_{xy} \\
\rho'_{yx} & \rho'_{yy}
\end{pmatrix}, \tag{4.15}
\]

To simplify the notation we shall drop the prime sign. However, one should bear in mind that all the formulae are written down in the coordinate system connected with the wavevector \( \mathbf{k} \), which is always parallel to the normal to the plate. The counterpart of equation (4.6) for the magnetic induction of the helicon \( \mathbf{b} \) takes the form:

\[
\mathbf{b} = \left(i c^2/4 \pi \omega \right) k^2 \rho_{xy} \hat{\mathcal{H}} \mathbf{b} \tag{4.16}
\]

where \( \hat{\mathcal{H}} \) is a two-dimensional tensor with components

\[
\mathcal{H}_{ik} = \begin{pmatrix}
\alpha_x \mu_{xx} + \mu_{xy} & \alpha_y \mu_{xy} + \mu_{yy} \\
\alpha_x \mu_{xy} - \mu_{xx} & \alpha_y \mu_{yy} - \mu_{xy}
\end{pmatrix}, \tag{4.17}
\]

Here \( \alpha_x = \rho_{xx}/\rho_{xy}, \alpha_y = \rho_{yy}/\rho_{xy} \).

The resulting dispersion relation for electromagnetic waves under the de Haas–van Alphen effect conditions takes the form (Volskii 1975):

\[
k^2_n = \pm \{4 \pi \omega [c^2 (\det \hat{\rho})^{1/2} (\det \hat{\mathbf{p}})^{1/2}]^{-1} \} \exp (\mp \varphi_k). \tag{4.18}
\]

It contains the extra oscillating factor \( (\det \hat{\mathbf{p}})^{1/2} \). The damping of the wave depends on the quantity \( \varphi_k \):

\[
\varphi_k = \sin^{-1} \left[ (\alpha_x \mu_{yy} + \alpha_y \mu_{xx}) \rho_{xy} / 2 (\det \hat{\rho})^{1/2} (\det \hat{\mathbf{p}})^{1/2} \right] \quad 0 \leq \varphi_k \leq \pi / 2. \tag{4.19}
\]

The polarisation of the wave, i.e. the ratio of the components of \( \mathbf{b} \), is given by

\[
f_\pm = f_0 \exp (\pm \varphi_f) \quad \varphi_f = \cos^{-1} \left[ (\mathcal{H}_{yy} - \mathcal{H}_{xx}) / 2 |\mathcal{H}_{xy}\mathcal{H}_{yx}|^{1/2} \right] \quad 0 \leq \varphi_f \leq \pi \tag{4.20}
\]

\[
f_0 = \left( \mathcal{H}_{xx} \mathcal{H}_{yy} \right)^{1/2}. \tag{4.20}
\]

Hence the magnetic induction wave of unit amplitude in the infinite metal slab can be
written as
\[ b_x = (1, f_x, 0) \exp \left[ i(\omega t - k_x z) \right]. \] (4.21)
The sign ' + ' corresponds to a weakly damped helicon. For the damped wave \( k_x = -k_x^e \).

The recorded signal in the secondary coil (2.87), using the method of crossed coils, for an arbitrary direction of the excitation field in the anisotropic case is as follows:

\[
V_T^2 = \frac{8i\omega}{\pi^2c} \frac{2d}{\alpha_x \mu_{yy} + \alpha_y \mu_{xx}} \left[ \left( 1 - \frac{\mu_{xy}}{\det \mu} (\alpha_x \mu_{yy} + \alpha_y \mu_{xx}) (\varphi_x^2 - \varphi_y^2) \right. \right.
\]
\[ + \frac{\mu_{xy}^2}{\det \mu} (\alpha_x - \alpha_y) \varphi_x \varphi_y - \frac{\alpha_x \mu_{xy}^2 - \alpha_y \mu_{xx}^2}{\det \mu} \varphi_x \varphi_y \sum_{n=1}^{\infty} (2n-1)^{-2} \]
\[ \times \left[ 1 + iQ(\omega/\omega_n - \omega_n/\omega) \right]^{-1} + \frac{(\det \mu)^{1/2}}{\rho_{xy}(\det \mu)^{1/2}} \]
\[ \times \left[ (\mu_{xy} (\varphi_x^2 - \varphi_y^2) - (\mu_{xx} - \mu_{yy}) \varphi_x \varphi_y) \sum_{n=1}^{\infty} (2n-1)^{-2} \right. \]
\[ \times (\omega_n/\omega)[1 + iQ(\omega/\omega_n - \omega_n/\omega)]^{-1} \right]. \] (4.22)

Here \( \varphi_x \) and \( \varphi_y \) are components of the unit vector in the direction of the excitation field: \( \mathbf{h}_0 = (\varphi_x, \varphi_y, 0) \exp (i\omega t) \). Resonance frequency and \( Q \) factor are invariant under rotation of the axes of the crossed coils:

\[
\omega_n = (2n-1)^2 (\pi/d)^2 (c^2/16\pi)(\det \hat{\mu})^{1/2} (\rho_{xy}(\det \hat{\mu})^{1/2})
\]
\[
Q = \frac{(\det \hat{\mu})^{1/2}}{\text{Sp} \hat{\mu}} = (1 + \alpha_x \alpha_y)^{1/2} (\mu_{xx}\mu_{yy} - \mu_{xy}^2)^{1/2} (\alpha_x \mu_{yy} + \alpha_y \mu_{xx})^{-1}. \] (4.23)

Excitation of helicons in the plate should be accompanied by the following effects (see formulae (4.22) and (4.23)).

(a) Oscillations of the resonance frequency, already discussed in the preceding subsection (see equation (4.14)). In the system connected with the plate (\( \mathbf{OZ} \) axis along the normal), \( (\det \hat{\mu})^{1/2} \) in (4.23) takes the form (Volskii and Petrashov 1970):

\[
(\det \hat{\mu})^{1/2} = \left\{ 1 - 4\pi \left( \frac{\partial M_x}{\partial B_x} - \frac{\partial M_y}{\partial B_y} \right) + 16\pi^2 \left[ \frac{\partial^2 M_x}{\partial B_x^2} - \frac{\partial^2 M_x}{\partial B_y^2} \right] \right\}^{1/2}. \] (4.24)

Using (4.23) and (4.24), one easily gets (4.14) for the spherical FS.

(b) Oscillations in the \( Q \) factor and in the resonance peak height should also be observable. In view of (4.24)

\[
Q = (1 + \alpha_x \alpha_y)^{1/2} (\alpha_x + \alpha_y)^{-1} \left\{ 1 - 4\pi \left( \frac{\partial M_x}{\partial B_x} + \frac{\partial M_y}{\partial B_y} \right) \right.
\]
\[ + 16\pi^2 \left[ \frac{\partial M_x}{\partial B_x} \frac{\partial M_y}{\partial B_y} - \left( \frac{\partial M_x}{\partial B_y} \right)^2 \right] \right\}^{1/2} \left[ 1 - 2\pi \left( \frac{\partial M_x}{\partial B_x} + \frac{\partial M_y}{\partial B_y} \right) \right.
\]
\[ - 2\pi (\alpha_x - \alpha_y) \left( \frac{\partial M_x}{\partial B_x} - \frac{\partial M_y}{\partial B_y} \right) \right]^{-1}. \] (4.25)

The relative oscillations of \( Q \) are smaller than those of the phase velocity. Moreover, if the magnetic field is normal to the plate, and the normal vector coincides with a
high-order symmetry axis, then there are practically no oscillations of \(Q\). Therefore, oscillations in \(Q\) can be observed only under asymmetric geometry. The most sensitive technique for observing variations of \(Q\), including oscillations, is by measurement of the resonance peak amplitude in the constant field method or the amplitude of the signal in the helicon generator method. According to formulae (4.22)–(4.25), both quantities are proportional to \(Q\).

In the asymmetric case one can also observe oscillations in the shape of the resonance curve, caused by the last term in (4.22) being shifted in phase by 90° relative to the first term.

5. Experimental study of quantum effects and their influence on helicon resonance

5.1. Quantum oscillations of the phase velocity of helicons

5.1.1. Method of measuring the absolute amplitude of the de Haas–van Alphen effect.

A typical helicon resonance curve recorded in indium under quantisation conditions is shown in figure 17(b). The correspondence with the graphical procedure of figure 17(a) is so exact that the interpretation of the effect, developed in the above sections, can hardly be questioned. The fine structure of resonances, observed in aluminium at large amplitude of the de Haas–van Alphen effect (region D–D' in figure 17(a)), is shown in figure 17(c). The oscillation amplitude \(\partial M/\partial B\) is enough to cause such large variations of the helicon wavelength (see (4.10) and (4.18)) that at extrema of \(\partial M/\partial B\) the resonance effect disappears altogether. One observes a sequence of resonance peaks following each other with the doubled frequency of the de Haas–van Alphen effect, since the resonance criterion (2.86) is fulfilled twice per oscillation period. A similar pattern is observed when the amplitude of the wavelength oscillations is large enough:

\[
\Delta k/k = \pi (\partial M/\partial B) \sin^2 \theta > 1/Q = (\omega_e \tau \cos \theta)^{-1}.
\]  

Oscillations depicted in figures 17(b) and (c) are induced by electrons on extremal cross sections of the FS of aluminium and indium in the third Brillouin zone (e.g. Cracknell and Wong 1973).

Of particular interest are the quantum oscillations of the helicon generator frequency induced when the magnetic field varies (see equations (4.14) and (4.23)). If the quantity \(4\pi (\partial M/\partial B) \sin^2 \theta\) is small enough, the relative amplitude of the oscillations of helicon generator frequency \(\Delta \omega/\omega\) is related to the amplitude of the de Haas–van Alphen oscillations \(\partial M/\partial B\) by a simple relationship:

\[
\Delta \omega/\omega = 2\pi (\partial M/\partial B) \sin^2 \theta
\]  

(similar relationships can be derived from (4.23) and (4.24) for the anisotropic case). We note particularly that neither the circuit parameters (the relative volume of the coil filled by the sample, the number of turns in coils, signal amplitude, etc), nor the sample magnetoelasticity, enter (5.2). Thus a helicon generator can be used for an absolute measurement of the de Haas–van Alphen effect amplitude (Volskii and Petrashov 1968, 1970). Angles in (5.2) and (4.24) can be determined, for example, from the helicon resonance position and from the frequency of the quantum oscillations. Precision measurements of the helicon generator are straightforward.
The above statements are valid for an arbitrary amplitude $\partial M / \partial B$, after (5.2) is substituted by a more exact equation:

$$\omega_{osc}/\omega_n = [1 - 4\pi(\partial M / \partial B) \sin^2 \theta]^{1/2}. \quad (5.3)$$

The 'classical' frequency $\omega_n$ can be measured, for example, by increasing the sample temperature. In the anisotropic case one should use formulae (4.23) and (4.24).

In the Lifshitz-Kosevich theory the fundamental oscillatory term of $\partial M / \partial B$, where $M$ is given by equation (4.3), can be written as

$$\frac{\partial M}{\partial B} = A \frac{T \exp \left(-2\pi^2 K X / h\omega_c\right)}{B^{5/2} \sinh \left(2\pi^2 K T / h\omega_c\right)} \cos \left[2\pi(F/B) + \varphi_1\right] \quad (5.4)$$

where

$$A = \left(2e/hc\right)^{5/2} \pi^{1/2} K F^2 \cos \left(\pi g_{\text{eff}} m_c / 2m_0\right) \hat{r}^2 S / \partial p_3^2 \right]^{-1/2}. \quad (5.5)$$

Here $g_{\text{eff}}$ is the effective electron $g$ factor and $X$ is the Dingle temperature, which characterises the collision broadening of the Landau levels.

According to (5.14) oscillations can be observed provided that $K(X + T) < h\omega_c$. This condition is much less restrictive than that of observation of quantum oscillations in Landau damping of helicons: $h\omega_c > \left[\varepsilon_f(h\tau^{-1} + KT)\right]^{1/2} / k_z l$ (Kaner and Skobov 1968).

For the scattering of electrons by point scatterers the Dingle temperature is related to the collision rate $\tau^{-1}$ via (e.g. Springford 1980)

$$X = \left(h / 2\pi K\right) \tau^{-1}. \quad (5.6)$$

Formulae (5.4)-(5.6) nicely illustrate the potentialities of the de Haas-van Alphen effect for determining many important parameters of electrons in metals. Oscillation frequency measurements are experimentally the easiest ones to make and are employed as one of the most accurate tools for measuring the areas of FS cross sections (e.g. Shoenberg 1969, Cracknell and Wong 1973). Measurements of the de Haas-van Alphen effect amplitude as a function of temperature and magnetic field enable one to determine such important quantities as the cyclotron mass $m_c$ and the collision broadening of Landau levels (5.6) (see, for example, Springford (1980)). Measurements of the absolute amplitude of the effect give additional possibilities for the determination of effective $g$ factors (Shoenberg and Vanderkooy 1970, Alles et al 1975), and information on spin-dependent scattering can be deduced from the harmonic content of the oscillations (4.3) (e.g. Alles et al 1975). The helicon generator method can be used for all the experiments in metals, in which helicons can propagate (see §3.1).

The experimentally recorded helicon frequencies of an indium helicon generator at temperatures 4.2 and 1.2 K are shown in figure 17(d). Lowering the temperature one encounters quantum oscillations due to the de Haas-van Alphen effect. There is complete agreement between the observed oscillations of the helicon generator and the formulae (4.23) and (4.24) and independent determinations of the FS parameters (Hughes and Shepherd 1969).

5.1.2. Helicon studies of defects generated during plastic deformation. Both collisional damping of helicons (2.75) and collision broadening of Landau levels (5.6) are due to the interaction of conduction electrons with impurities, lattice imperfections and phonons. However, mechanisms of transport relaxation and of the broadening of levels in a magnetic field differ, making their sensitivity to defects of different kinds distinct
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(Gantmakher et al 1972, Terwilliger and Higgins 1973). One of the main distinctions is due to the extremely high sensitivity of the Dingle temperature to small-angle scattering. This makes measurements of the amplitude of quantum oscillations and of the damping of helicons complementary, thus enabling one to deduce information not only on the scattering of conduction electrons, but also on the defects.

As an example of such simultaneous measurement of the absolute amplitude of quantum oscillations and of the helicon damping we cite the works of Volskii et al.

![Figure 18. Damping of helicons and the de Haas-van Alphen effect during low-temperature plastic deformation in aluminum.](image)

Figure 18. Damping of helicons and the de Haas-van Alphen effect during low-temperature plastic deformation in aluminium. (a) ●, The Dingle temperature \( \chi \); ○, the damping of helicons in the form \( \rho_H = \Gamma B_0 R_H \) (\( R_H = 1.02 \times 10^{-15} \Omega \text{ cm G}^{-1} \)); *, the relative sample elongation as a function of uniaxial tension at 1.3 K; (b) the helicon resonance and the de Haas-van Alphen effect amplitude at different stages: (I) before the deformation, (II) after 2% elongation, (III) after annealing at 140 °C, (IV) after annealing at 500 °C. The resonance was recorded at \( B_0 = 7 \text{ kG} \), the oscillations were recorded at \( B_0 = 13.5 \text{ kG} \) (from Volskii et al 1973).
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(1973) and Petrashov (1978), who made measurements directly in the course of experiments on the deformation of aluminium at liquid helium temperatures†. The helicon damping and the Dingle temperature $X$, determined from the absolute amplitude of quantum oscillations, are shown in figure 18(a).

For stresses in the elastic deformation region in figure 18(a) the Dingle temperature and the helicon damping are unaffected. With the onset of plastic deformation the Landau levels broaden and the transport collision rate $\tau_{\text{eff}}^{-1}$ deduced from the helicon damping rises. The reason is straightforward: new defects, dislocations and vacancies are generated by the plastic deformation, thus enhancing the electron collision rate and causing the increase of $X$ and $\tau_{\text{eff}}^{-1}$, whereas in the Hooke region there is practically no generation of new defects.

The most interesting observation is that the rise of $X$ and $\tau_{\text{eff}}^{-1}$ is caused by different defects. The rise of the Dingle temperature is dominated by dislocations, whereas vacancies dominate in the transport collision rate. This was ascertained by an annealing procedure. Changes of the measured characteristics of the metal are shown qualitatively in figure 18(b). Also shown are the helicon resonance and quantum oscillations before and after the deformation, after annealing at 140 °C, which eliminates vacancies generated by deformation, and after annealing at 500 °C, which eliminates dislocations (Federighi 1965, Nøst and Sørensen 1968). The first annealing at 140 °C restores the $Q$ factor of the helicon resonance, whereas the de Haas–van Alphen effect amplitude remains at the noise level. Annealing at 500 °C practically does not affect the helicon $Q$ factor, while restoring the initial pre-deformation amplitude of quantum oscillations. One can further lower the density of dislocations in a sample using special annealing methods (see Nøst 1965, Nes and Nøst 1966). This results in an enhancement of the quantum oscillation amplitude by a few times compared with the pre-deformation amplitude, while causing practically no change of the helicon $Q$ factor (Petrashov 1978).

Analysing the results one concludes that simultaneous measurements of the collision damping of helicons and of the broadening of Landau levels enable one to study separately the generation of dislocations and vacancies in the course of plastic deformation at low temperatures (for details see Volskii et al (1973) and Petrashov (1978)). Such measurements may be useful in studies of the mechanisms of creation of defects during low-temperature deformation of metals.

5.2. Non-linear helicon resonance

All the above considerations refer to a linear approximation in the magnetic induction $b$ of the wave. In particular, the $h$–$b$ relationship was taken to be linear in equation (4.5).

However, a magnetic induction $b$ of the wave may well become comparable to the period of oscillation of the magnetic moment. If so, one encounters the non-linear problem and the exact non-linear function $M(B)$ should be used, and all the parameters of the wave, such as phase velocity, damping, polarisation, etc, become non-linear functions of the changing magnetic induction. The non-linearity parameter is the ratio of the amplitude of helicon induction $b_0$ to the period $\Delta B$ of oscillations in the de Haas–van Alphen effect. A convenient definition is

$$
\varepsilon = \frac{2\pi b_0}{\Delta B} = \frac{2\pi F b_0}{B^2} = 2\pi n \left( \frac{b_0}{B} \right).
$$

(5.7)

† The first studies of the influence of low-temperature deformation on the damping of helicons are due to Penz and Bowers (1968) (in potassium).
For metals $F = 10^6-10^8$ and the non-linearity parameter $\epsilon$ reaches unity if $b_0 \approx 1\text{G}$ at fields $B_0 = 10^4\text{G}$. In resonance, the in-sample field of the wave is larger by a factor of $Q$ than the excitation field of the coil. Clearly it is extremely easy to obtain the non-linear regime conditions. The parameter $\epsilon$ is not readily expressed in terms of the amplitude of $(\partial M/\partial B)$, although the magnitude of the non-linear effects naturally depends on it.

Bozhko and Volskii (1977a) were the first to observe the non-linear helicon resonance (in aluminium). It manifested itself in variations of the position of the standing wave resonances and the shape of resonance lines as a function of the wave magnetic field, including the hysteresis-like frequency dependence of the resonance pattern when the signal is recorded in a fixed external field. The crossed-coils technique was used.

The helicon generator frequency was recorded with a weak excitation field (figure 19(a)). Then the magnetic field was kept fixed at a few positions along the oscillation peak and the frequency dependence of the signal was recorded in each case for a few amplitudes of the excitation field (figure 19(b)). The observed non-linear effect is largest at the oscillation peak maximum or minimum (points 1 and 2 in figure 19(a)). In this case the increase of the excitation field shifts the resonance maximum towards the position of vanishing amplitude of the de Haas–van Alphen effect, makes one of the resonance wings steeper and introduces hysteresis into the signal when the recording direction is altered. When the estimated amplitude of the in-metal helicon field far exceeds the de Haas–van Alphen oscillation period, the resonance curve shifts to its zero de Haas–van Alphen amplitude position and restores its symmetry. At type-3 points in figure 19(a) there is practically no non-linear effect. Such a behaviour was observed both with oscillations caused by the electrons in the third zone of the $\text{FS}$ and the holes in the second zone.

The non-linear resonance of helicons has a counterpart in the behaviour of oscillators containing non-linear elements which one is familiar with in mechanics and electronics. A complete theory of non-linear helicon resonance does not yet exist. However, the initial stages, when quadratic expansion of $\textbf{M}$ in terms of $\textbf{b}$ is sufficient, can be described quantitatively (Bozhko and Volskii 1977a).
5.3. Helicon resonance in the presence of diamagnetic domains

The magnetisation $M$, magnetic field $H$ in the sample and the mean microscopic field (inductance) $B$ are related by

$$H(B) = B - 4\pi M(B).$$  \hspace{1cm} (5.8)

Since $H$ is a function of $M$, oscillations due to the de Haas-van Alphen effect may result in such values of $H$, for which (5.8) gives three solutions for $B$ (figure 20(a)). Such an ambiguity signals instability of the state, since by virtue of the thermodynamic constraint $\partial H/\partial B > 0$ the region AB in figure 20(a) is forbidden (Shoenberg 1962). This instability results in a phase transition when the parallel-plate sample acquires a domain structure with different inductions $B_1$ and $B_2$ inside domains (Condon and Volskii 1968).

The influence of the transition into the diamagnetic domain state upon the propagation of helicons was studied by Bozhko and Volskii (1977b) in aluminium. In order to create the thermodynamic instability one needs $4\pi (\partial M/\partial B)$ to be of the order of unity. It took some effort to prepare samples of adequate perfection. The commonly used method of growing crystals from the melt in vacuum, using quartz or graphite moulds, proved to be unsuitable since it produced samples with Dingle temperatures of the order of 0.5 K. Besides this, the crystals proved to have a mosaic spread, thus producing a parasitic splitting of the helicon resonance (Bozhko and Volskii 1975).

To get the required large amplitude of the de Haas-van Alphen effect, a novel technique of producing perfect aluminium crystals, first employed in studies of low-temperature electron properties of metals by Petrashov (1974), was used. This method consisted of a sequence of cold work and annealing, worked out using the results of recrystallisation studies in aluminium by Nøst (1965) and Nes and Nøst (1966). This yielded samples with Dingle temperatures of the order of 0.05 K, the measurements being carried out at temperatures below 1 K using a $^3$He cryostat (Lounasmaa 1974).

The influence of a transition to the domain state on helicons was studied by recording the resonance curves as a function of frequency at different temperatures. An external magnetic field was kept fixed at different points of one oscillation peak. In figures 20(b) and (c) (Bozhko and Volskii 1977a, b), we show the maximum and the minimum of the oscillating resonance frequency and the relative damping. Also shown in the
plot of frequencies are curves calculated by substituting (5.4) into (4.14) and the horizontal straight line, marking $f_{\text{res}}$ at $4\pi(\partial M/\partial B) = 1$, indicating the point of transition into the domain state.

One sees that the observed maximum of the resonance follows the temperature dependence dictated by (4.14), whereas the minimum frequency tends to the value corresponding to the domain state and then depends weakly on temperature. The dashed part of the curve in figure 20(b) denotes the thermodynamically forbidden region of helicon resonance frequencies.

One more important feature of the results of Bozhko and Volskii (1977b) is a steep rise of the helicon damping after transition into the region of unstable uniform magnetisation.

The observed amplitudes of oscillations of the helicon resonance frequency agree with the theory developed in the preceding subsection. This tempts one to connect the peculiar behaviour of the maximum and minimum resonance frequencies with the formation of the domain structure in metal. However, the observed temperature dependence of the damping and the presence of additional damping after the domain formation were quite unexpected and require clarification.

The electrodynamics of helicons in the presence of diamagnetic domains has yet to be developed fully. A semiquantitative description of the phenomenon was given by Volskii (1977), who connected the damping of helicons with dissipative eddy currents, induced by motion of the domain walls in the oscillating field of helicons. Similar effects exist in the flipping of the magnetisation of metallic ferromagnetics (Williams et al 1950).

5.4. Quantum oscillations of helicon damping due to magnetic breakdown

As was discussed in § 2, the damping of helicons is extremely sensitive to the presence of magnetic breakdown and can be used as a sensitive tool for studying this phenomenon.

![Figure 21. Influence of magnetic breakdown on the damping of helicons in aluminium. (a) The damping of helicons in the form of the quantity $\rho_h = \frac{\Gamma B_0 R_{\text{H}}}{n}$ ($R_{\text{H}}$ is the Hall constant) as a function of magnetic field. $\times$, magnetic-breakdown-free case, $\bigcirc$, under conditions of the breakdown, $B_0[100]$ (Delaney 1974); (b) angular dependence of helicon damping under conditions of magnetic breakdown (Bozhko et al 1977). In the breakdown-free case the angular dependence is very weak in such a narrow angular range.](image-url)
The influence of magnetic breakdown on the damping of helicons in aluminium was studied by Delaney (1974) and Bozhko et al (1977) (see figures 21(a) and (b)). In the absence of magnetic breakdown the quantity $\Gamma R_{HI}B_0$, which is proportional to the transverse magnetoresistivity of aluminium, is practically independent of the magnetic field (figure 21(a), bottom curve). This signals that there are no open orbits for this particular direction of magnetic field. The oscillations in the helicon damping, induced by the magnetic breakdown, are also shown in figure 21(a) (the upper curve). Generation of open orbits in aluminium via magnetic breakdown is a complicated process involving the transition of electrons from the FS in the second Brillouin zone to that in the third zone and followed by a transition back to the second zone, etc (figure 2(d)). In such a situation the FS in the third zone acts essentially as a bridge. Oscillations arise as a result of an interference of electron waves in the magnetic breakdown region of the momentum space (Pippard 1965, Young 1971).

Magnetic-breakdown-induced oscillations in helicon propagation in tin have been studied by Hays and McLean (1968). The magnetic breakdown in indium was first observed employing the helicon technique (Petrashov and Springford 1983).

6. Conclusions

During the last decade helicon studies have become more commonplace as a wider range of super-pure metals in which they may be studied has become available. When dealing with metals in a magnetic field at low temperatures, one cannot ignore the possible presence of helicons, because their propagation can drastically affect not only the electrodynamical properties of metals but also the mechanical properties.

Progress in helicon studies has led to the development of a new technique of considerable utility which may find a useful application in the clarification of some unresolved problems, such as the mysterious anomalies of seemingly simple metals like potassium and thermodynamically unstable states at large amplitudes of the de Haas–van Alphen effect. In addition, helicon methods may prove very useful in studies of the defect structure of metals, which determine many of their important properties.

A number of features of the electrodynamics of helicons have yet to be studied. This is true, for example, in non-linear helicon resonance, one of the most strongly non-linear phenomena in the electrodynamics of metals. Its peculiarity is that in the propagation of helicons there is no skin effect, so that non-linear phenomena are a property of the bulk and the non-linearity parameter can readily be made large by a suitable choice of experimental conditions. Another area to be explored is the giant quantum oscillations of the Landau damping of helicons, although convincing evidence for their existence is as yet lacking. Finally, there is need for an exact solution for helicons propagating in finite-sized samples of metals having an anisotropic FS.

As regards future advances in the field, this will inevitably include an extension of experimental work to both higher magnetic fields and lower temperatures. Furthermore, the quality of samples must be improved to yield experimental results of a better quality and to enhance the amplitude of quantum effects.

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